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Market Size, Competition, and the Product Mix of Exporters*

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Abstract

We build a theoretical model of multi-product firms that highlights how market size and geography (the market sizes of and bilateral economic distances to trading partners) affect both a firm's exported product range and its exported product mix across market destinations (the distribution of sales across products for a given product range). We show how tougher competition in an export market induces a firm to skew its export sales towards its best performing products. We find very strong confirmation of this competitive effect for French exporters across export market destinations. Trade models based on exogenous markups cannot explain this strong significant link between destination market characteristics and the within-firm skewness of export sales (after controlling for bilateral trade costs). Theoretically, this within firm change in product mix driven by the trading environment has important repercussions on firm productivity and how it responds to changes in that trading environment.

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1 Introduction

Recent empirical evidence has highlighted how the export patterns of multi-product firms dominate world trade flows, and how these multi-product firms respond to different economic conditions across export markets by varying the number of products they export.¹ In this paper, we further analyze the effects of those export market conditions on the relative export sales of those goods: we refer to this as the firm’s product mix choice. We build a theoretical model of multi-product firms that highlights how market size and geography (the market sizes of and bilateral economic distances to trading partners) affect both a firm’s exported product range and its exported product mix across market destinations. We show how tougher competition in an export market – associated with a downward shift in the distribution of markups across all products sold in the market – induces a firm to skew its export sales towards its best performing products. We find very strong confirmation of this competitive effect for French exporters across export market destinations. Our theoretical model shows how this effect of export market competition on a firm’s product mix then translates into differences in measured firm productivity: when a firm skews its production towards better performing products, it also allocates relatively more workers to the production of those goods and raises its overall output (and sales) per worker. Thus, a firm producing a given set of products with given unit input requirements will produce relatively more output and sales per worker (across products) when it exports to markets with tougher competition. To our knowledge, this is a new channel through which competition (both in export markets and at home) affects firm-level productivity. This effect of competition on firm-level productivity is compounded by another channel that operates through the endogenous response of the firm’s product range: firms respond to increased competition by dropping their worst performing products.²

Feenstra and Ma (2008) and Eckel and Neary (2010) also build theoretical models of multi-product firms that highlight the effect of competition on the distribution of firm product sales. Both models incorporate the cannibalization effect that occurs as large firms expand their product range. In our model, we rely on the competition effects from the demand side, which are driven by variations in the number of sellers and their average prices across export markets. The cannibalization effect does not occur as a continuum of firms each produce a discrete number of products and

¹See Mayer and Ottaviano (2007) for Europe, Bernard et al (2007) for the U.S., and Arkolakis and Muendler (2010) for Brazil.

²Bernard et al (forthcoming) and Eckel and Neary (2010) emphasize this second channel. They show how trade liberalization between symmetric countries induces firms to drop their worst performing products (a focus on “core competencies”) leading to intra-firm productivity gains. We discuss those papers in further detail below.

thus never attain finite mass. The benefits of this simplification is that we can consider an open economy equilibrium with multiple asymmetric countries and asymmetric trade barriers whereas Feenstra and Ma (2008) and Eckel and Neary (2010) restrict their analysis to a single globalized world with no trade barriers. Thus, our model is able to capture the key role of geography in shaping differences in competition across export market destinations.³

Another approach to the modeling of multi-product firms relies on a nested C.E.S. structure for preferences, where a continuum of firms produce a continuum of products. The cannibalization effect is ruled out by restricting the nests in which firms can introduce new products. Allanson and Montagna (2005) consider such a model in a closed economy, while Arkolakis and Muendler (2010) and Bernard et al (forthcoming) develop extensions to open economies. Given the C.E.S. structure of preferences and the continuum assumptions, markups across all firms and products are exogenously fixed. Thus, differences in market conditions or proportional reductions in trade costs have no effect on a firm's product mix choice (the relative distribution of export sales across products). In contrast, variations in markups across destinations (driven by differences in competition) generate differences in relative exports across destinations in our model: a given firm selling the same two products across different markets will export relatively more of the better performing product in markets where competition is tougher. In our comprehensive data covering nearly all French exports, we find that there is substantial variation in this relative export ratio across French export destinations, and that this variation is consistently related to differences in market size and geography across those destinations (market size and geography both affect the toughness of competition across destinations).

Theoretically, we show how this effect of tougher competition in an export market on the exported product mix is also associated with an increase in productivity for the set of exported products to that market. We show how firm-level measures of exported output per worker as well as deflated sales per worker for a given export destination (counting only the exported units to a given destination and the associated labor used to produce those units) increase with tougher competition in that destination. This effect of competition on firm productivity holds even when one fixes the set of products exported, thus eliminating any potential effects from the extensive (product) margin of trade. In this case, the firm-level productivity increase is entirely driven by the response of the

³Nocke and Yeaple (2008) and Baldwin and Gu (2009) also develop models with multi-product firms and a pro-competitive effect coming from the demand side. These models investigate the effects of globalization on a firm's product scope and average production levels per product. However, those models consider the case of firms producing symmetric products whereas we focus on the effects of competition on the within-firm distribution of product sales.

firm's product mix: producing relatively more of the better performing products raises measured firm productivity. Our model also features a response of the extensive margin of trade: tougher competition in the domestic market induces firms to reduce the set of produced products, and tougher competition in an export market induces exporters to reduce the set of exported products. We do not emphasize these results for the extensive margin, because they are quite sensitive to the specification of fixed production and export costs. In order to maintain the tractability of our multi-country asymmetric open economy, we abstract from those fixed costs (increasing returns are generated uniquely from the fixed/sunk entry cost). Conditional on the production and export of given sets of products, such fixed costs would not affect the relative production or export levels of those products. These are the product mix outcomes that we emphasize (and for which we find strong empirical support).

Although we focus our empirical analysis on the cross-section of export destinations for French exporters, other studies have examined the effects of trade liberalization over time on the extensive and intensive margins of production and trade. Baldwin and Gu (2009), Bernard et al (forthcoming), and Iacovone and Javorcik (2008) all show how trade liberalization in North America induced (respectively) Canadian, U.S., and Mexican firms to reduce the number of products they produce. Baldwin and Gu (2009) and Bernard et al (forthcoming) further report that CUSFTA induced a significant increase in the skewness of production across products (an increase in entropy). This could be due to an extensive margin effect if it were driven by production increases for newly exported goods following CUSTA, or to an intensive margin effect if it were driven by the increased skewness of domestic and export sales (a product mix response). Iacovone and Javorcik (2008) report that this second channel was dominant for the case of Mexico. They show that Mexican firms expanded their exports of their better performing products (higher market shares) significantly more than those for their worse performing exported products during the period of trade expansion from 1994-2003. They also directly compare the relative contributions of the extensive and intensive product margins of Mexican firms' exports to the U.S.. They find that changes in the product mix explain the preponderance of the changes in the export patterns of Mexican firms. Arkolakis and Muendler (2010) find a similar result for the export patterns of Brazilian firms to the U.S.: Because the firms' exported product mix is so skewed, changes at the extensive margin contribute very little to a firm's overall exports (the newly exported products have very small market shares relative to the better performing products previously exported).

Our paper proceeds as follows. We first develop a closed economy version of our model in

order to focus on the endogenous responses of a firm's product scope and product mix to market conditions. We highlight how competition affects the skewness of a firm's product mix, and how this translates into differences in firm productivity. Thus, even in a closed economy, increases in market size lead to increases in within-firm productivity via this product mix response. We then develop the open economy version of our model with multiple asymmetric countries and an arbitrary matrix of bilateral trade costs. The equilibrium connects differences in market size and geography to the toughness of competition in every market, and how the latter shapes a firm's exported product mix to that destination. We then move on to our empirical test for this exported product mix response for French firms. We show how destination market size as well as its geography induce increased skewness in the firms' exported product mix to that destination.

2 Closed Economy

Our model is based on an extension of Melitz and Ottaviano (2008) that allows firms to endogenously determine the set of products that they produce. We start with a closed economy version of this model where L consumers each supply one unit of labor.

2.1 Preferences and Demand

Preferences are defined over a continuum of differentiated varieties indexed by $i \in \Omega$, and a homogenous good chosen as numeraire. All consumers share the same utility function given by

$$U = q_0^c + \alpha \int_{i \in \Omega} q_i^c di - \frac{1}{2} \gamma \int_{i \in \Omega} (q_i^c)^2 di - \frac{1}{2} \eta \left(\int_{i \in \Omega} q_i^c di \right)^2, \quad (1)$$

where q_0^c and q_i^c represent the individual consumption levels of the numeraire good and each variety i . The demand parameters α , η , and γ are all positive. The parameters α and η index the substitution pattern between the differentiated varieties and the numeraire: increases in α and decreases in η both shift out the demand for the differentiated varieties relative to the numeraire. The parameter γ indexes the degree of product differentiation between the varieties. In the limit when $\gamma = 0$, consumers only care about their consumption level over all varieties, $Q^c = \int_{i \in \Omega} q_i^c di$, and the varieties are then perfect substitutes. The degree of product differentiation increases with γ as consumers give increasing weight to smoothing consumption levels across varieties.

The marginal utilities for all varieties are bounded, and a consumer may not have positive demand for any particular variety. We assume that consumers have positive demand for the numeraire

good ($q_0^c > 0$). The inverse demand for each variety i is then given by

$$p_i = \alpha - \gamma q_i^c - \eta Q^c, \quad (2)$$

whenever $q_i^c > 0$. Let $\Omega^* \subset \Omega$ be the subset of varieties that are consumed (such that $q_i^c > 0$). (2) can then be inverted to yield the linear market demand system for these varieties:

$$q_i \equiv Lq_i^c = \frac{\alpha L}{\eta M + \gamma} - \frac{L}{\gamma} p_i + \frac{\eta M}{\eta M + \gamma} \frac{L}{\gamma} \bar{p}, \quad \forall i \in \Omega^*, \quad (3)$$

where M is the measure of consumed varieties in Ω^* and $\bar{p} = (1/M) \int_{i \in \Omega^*} p_i di$ is their average price. The set Ω^* is the largest subset of Ω that satisfies

$$p_i \leq \frac{1}{\eta M + \gamma} (\gamma \alpha + \eta M \bar{p}) \equiv p^{\max}, \quad (4)$$

where the right hand side price bound p^{\max} represents the price at which demand for a variety is driven to zero. Note that (2) implies $p^{\max} \leq \alpha$. In contrast to the case of C.E.S. demand, the price elasticity of demand, $\varepsilon_i \equiv |(\partial q_i / \partial p_i) (p_i / q_i)| = [(p^{\max} / p_i) - 1]^{-1}$, is not uniquely determined by the level of product differentiation γ . Given the latter, lower average prices \bar{p} or a larger number of competing varieties M induce a decrease in the price bound p^{\max} and an increase in the price elasticity of demand ε_i at any given p_i . We characterize this as a ‘tougher’ competitive environment.⁴

Welfare can be evaluated using the indirect utility function associated with (1):

$$U = I^c + \frac{1}{2} \left(\eta + \frac{\gamma}{M} \right)^{-1} (\alpha - \bar{p})^2 + \frac{1}{2} \frac{M}{\gamma} \sigma_p^2, \quad (5)$$

where I^c is the consumer’s income and $\sigma_p^2 = (1/M) \int_{i \in \Omega^*} (p_i - \bar{p})^2 di$ represents the variance of prices. To ensure positive demand levels for the numeraire, we assume that $I^c > \int_{i \in \Omega^*} p_i q_i^c di = \bar{p} Q^c - M \sigma_p^2 / \gamma$. Welfare naturally rises with decreases in average prices \bar{p} . It also rises with increases in the variance of prices σ_p^2 (holding the mean price \bar{p} constant), as consumers then re-optimize their purchases by shifting expenditures towards lower priced varieties as well as the numeraire good.⁵ Finally, the demand system exhibits ‘love of variety’: holding the distribution of prices constant

⁴We also note that, given this competitive environment (given N and \bar{p}), the price elasticity ε_i monotonically increases with the price p_i along the demand curve.

⁵This welfare measure reflects the reduced consumption of the numeraire to account for the labor resources used to cover the entry costs.

(namely holding the mean \bar{p} and variance σ_p^2 of prices constant), welfare rises with increases in product variety M .

2.2 Production and Firm Behavior

Labor is the only factor of production and is inelastically supplied in a competitive market. The numeraire good is produced under constant returns to scale at unit cost; its market is also competitive. These assumptions imply a unit wage. Entry in the differentiated product sector is costly as each firm incurs product development and production startup costs. Subsequent production of each variety exhibits constant returns to scale. While it may decide to produce more than one variety, each firm has one key variety corresponding to its ‘core competency’. This is associated with a core marginal cost c (equal to unit labor requirement).⁶ Research and development yield uncertain outcomes for c , and firms learn about this cost level only after making the irreversible investment f_E required for entry. We model this as a draw from a common (and known) distribution $G(c)$ with support on $[0, c_M]$.

The introduction of an additional variety pulls a firm away from its core competency. This entails incrementally higher marginal costs of production for those varieties. The divergence from a firm’s core competency may also be reflected in diminished product quality/appeal. For simplicity, we maintain product symmetry on the demand side and capture any decrease in product appeal as an increased production cost. We refer to this incremental production cost as a customization cost.

A firm can introduce any number of new varieties, but each additional variety entails an additional customization cost (as firms move further away from their core competency). We index by m the varieties produced by the same firm in increasing order of distance from their core competency $m = 0$ (the firm’s core variety). We then denote $v(m, c)$ the marginal cost for variety m produced by a firm with core marginal cost c and assume $v(m, c) = \omega^{-m}c$ with $\omega \in (0, 1)$. This defines a firm-level ‘competence ladder’ with geometrically increasing customization costs. In the limit, as ω goes to zero, any operating firm will only produce its core variety and we are back to single product firms as in Melitz and Ottaviano (2008).

Since the entry cost is sunk, firms that can cover the marginal cost of their core variety survive and produce. All other firms exit the industry. Surviving firms maximize their profits using the

⁶We use the same concept of a firm’s core competency as Eckel and Neary (2010). For simplicity, we do not model any overhead production costs. This would significantly increase the complexity of our model without yielding much new insight.

residual demand function (3). In so doing, those firms take the average price level \bar{p} and total number of varieties M as given. This monopolistic competition outcome is maintained with multi-product firms as any firm can only produce a countable number of products, which is a subset of measure zero of the total mass of varieties M .

The profit maximizing price $p(v)$ and output level $q(v)$ of a variety with cost v must then satisfy

$$q(v) = \frac{L}{\gamma} [p(v) - v]. \quad (6)$$

The profit maximizing price $p(v)$ may be above the price bound p^{\max} from (4), in which case the variety is not supplied. Let v_D reference the cutoff cost for a variety to be profitably produced. This variety earns zero profit as its price is driven down to its marginal cost, $p(v_D) = v_D = p^{\max}$, and its demand level $q(v_D)$ is driven to zero. Let $r(v) = p(v)q(v)$, $\pi(v) = r(v) - q(v)v$, $\lambda(v) = p(v) - v$ denote the revenue, profit, and (absolute) markup of a variety with cost v . All these performance measures can then be written as functions of v and v_D only:⁷

$$\begin{aligned} p(v) &= \frac{1}{2} (v_D + v), \\ \lambda(v) &= \frac{1}{2} (v_D - v), \\ q(v) &= \frac{L}{2\gamma} (v_D - v), \\ r(v) &= \frac{L}{4\gamma} [(v_D)^2 - v^2], \\ \pi(v) &= \frac{L}{4\gamma} (v_D - v)^2. \end{aligned} \quad (7)$$

The threshold cost v_D thus summarizes the competitive environment for the performance measures of all produced varieties. As expected, lower cost varieties have lower prices and earn higher revenues and profits than varieties with higher costs. However, lower cost varieties do not pass on all of the cost differential to consumers in the form of lower prices: they also have higher markups (in both absolute and relative terms) than varieties with higher costs.

Firms with core competency $v > v_D$ cannot profitably produce their core variety and exit. Hence, $c_D = v_D$ is also the cutoff for firm survival and measures the ‘toughness’ of competition in the market: it is a sufficient statistic for all performance measures across varieties and firms.⁸ We

⁷Given the absence of cannibalization motive, these variety level performance measures are identical to the single product case studied in Melitz and Ottaviano (2008). This tractability allows us to analytically solve the closed and open equilibria with heterogenous firms (and asymmetric countries in the open economy).

⁸We will see shortly how the average price of all varieties and the number of varieties is uniquely pinned-down by

assume that c_M is high enough that it is always above c_D , so exit rates are always positive. All firms with core cost $c < c_D$ earn positive profits (gross of the entry cost) on their core varieties and remain in the industry. Some firms will also earn positive profits from the introduction of additional varieties. In particular, firms with cost c such that $v(m, c) \leq v_D \iff c \leq \omega^m c_D$ earn positive profits on their m -th *additional* variety and thus produce at least $m + 1$ varieties. The total number of varieties produced by a firm with cost c is

$$M(c) = \begin{cases} 0 & \text{if } c > c_D, \\ \max \{m \mid c \leq \omega^m c_D\} + 1 & \text{if } c \leq c_D. \end{cases} \quad (8)$$

which is (weakly) decreasing for all $c \in [0, c_M]$. Accordingly, the number of varieties produced by a firm with cost c is indeed an integer number (and not a mass with positive measure). This number is an increasing step function of the firm's productivity $1/c$, as depicted in Figure 1 below. Firms with higher core productivity thus produce (weakly) more varieties.

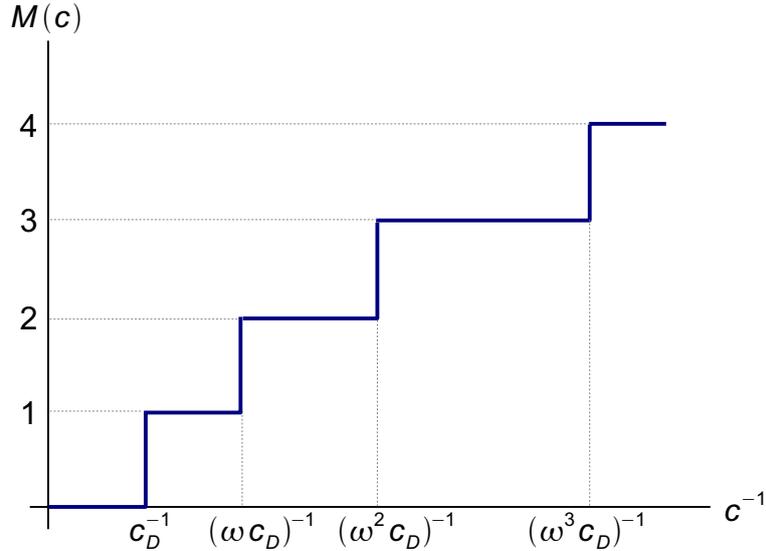


Figure 1: Number of Varieties Produced as a Function of Firm Productivity

Given a mass of entrants N_E , the distribution of costs across all varieties is determined by the optimal firm product range choice $M(c)$ as well as the distribution of core competencies $G(c)$. Let $M_v(v)$ denote the measure function for varieties (the measure of varieties produced at cost v or lower, given N_E entrants). Further define $H(v) \equiv M_v(v)/N_E$ as the normalized measure of varieties this cutoff.

per unit mass of entrants. Then $H(v) = \sum_{m=0}^{\infty} G(\omega^m v)$ and is exogenously determined from $G(\cdot)$ and ω . Given a unit mass of entrants, there will be a mass $G(v)$ of varieties with cost v or less; a mass $G(\omega v)$ of first additional varieties (with cost v or less); a mass $G(\omega^2 v)$ of second additional varieties; and so forth. The measure $H(v)$ sums over all these varieties.

2.3 Free Entry and Equilibrium

Prior to entry, the expected firm profit is $\int_0^{c_D} \Pi(c) dG(c) - f_E$ where

$$\Pi(c) \equiv \sum_{m=0}^{M(c)-1} \pi(v(m, c)) \quad (9)$$

denotes the profit of a firm with cost c . If this profit were negative for all c 's, no firms would enter the industry. As long as some firms produce, the expected profit is driven to zero by the unrestricted entry of new firms. This yields the equilibrium free entry condition:

$$\begin{aligned} \int_0^{c_D} \Pi(c) dG(c) &= \int_0^{c_D} \left[\sum_{\{m | \omega^{-m} c \leq c_D\}} \pi(\omega^{-m} c) \right] dG(c) \\ &= \sum_{m=0}^{\infty} \left[\int_0^{\omega^m c_D} \pi(\omega^{-m} c) dG(c) \right] = f_E, \end{aligned} \quad (10)$$

where the second equality first averages over the m^{th} produced variety by all firms, then sums over m .

The free entry condition (10) determines the cost cutoff $c_D = v_D$. This cutoff, in turn, determines the aggregate mass of varieties, since $v_D = p(v_D)$ must also be equal to the zero demand price threshold in (4):

$$v_D = \frac{1}{\eta M + \gamma} (\gamma \alpha + \eta M \bar{p}).$$

The aggregate mass of varieties is then

$$M = \frac{2\gamma}{\eta} \frac{\alpha - v_D}{v_D - \bar{v}}, \quad (11)$$

where the average cost of all varieties

$$\bar{v} = \frac{1}{M} \int_0^{v_D} v dM_v(v) = \frac{1}{N_E H(v_D)} \int_0^{v_D} v N_E dH(v) = \frac{1}{H(v_D)} \int_0^{v_D} v dH(v)$$

depends only on v_D .⁹ Similarly, this cutoff also uniquely pins down the average price across all varieties:

$$\bar{p} = \frac{1}{M} \int_0^{v_D} p(v) dM_v(v) = \frac{1}{H(v_D)} \int_0^{v_D} p(v) dH(v).$$

Finally, the mass of entrants is given by $N_E = M/H(v_D)$, which can in turn be used to obtain the mass of producing firms $N = N_E G(c_D)$.

2.4 Parametrization of Technology

All the results derived so far hold for any distribution of core cost draws $G(c)$. However, in order to simplify some of the ensuing analysis, we use a specific parametrization for this distribution. In particular, we assume that core productivity draws $1/c$ follow a Pareto distribution with lower productivity bound $1/c_M$ and shape parameter $k \geq 1$. This implies a distribution of cost draws c given by

$$G(c) = \left(\frac{c}{c_M} \right)^k, \quad c \in [0, c_M]. \quad (12)$$

The shape parameter k indexes the dispersion of cost draws. When $k = 1$, the cost distribution is uniform on $[0, c_M]$. As k increases, the relative number of high cost firms increases, and the cost distribution is more concentrated at these higher cost levels. As k goes to infinity, the distribution becomes degenerate at c_M . Any truncation of the cost distribution from above will retain the same distribution function and shape parameter k . The productivity distribution of surviving firms will therefore also be Pareto with shape k , and the truncated cost distribution will be given by $G_D(c) = (c/c_D)^k$, $c \in [0, c_D]$.

When core competencies are distributed Pareto, then all produced varieties will share the same Pareto distribution:

$$H(c) = \sum_{m=0}^{\infty} G(\omega^m c) = \Omega G(c), \quad (13)$$

where $\Omega = (1 - \omega^k)^{-1} > 1$ is an index of multi-product flexibility (which varies monotonically with ω). In equilibrium, this index will also be equal to the average number of products produced across all surviving firms:

$$\frac{M}{N} = \frac{H(v_D) N_E}{G(c_D) N_E} = \Omega.$$

The Pareto parametrization also yields a simple closed-form solution for the cost cutoff c_D from

⁹We also use the relationship between average cost and price $\bar{v} = 2\bar{p} - v_D$, which is obtained from (7).

the free entry condition (10):

$$c_D = \left[\frac{\gamma \phi}{L\Omega} \right]^{\frac{1}{k+2}}, \quad (14)$$

where $\phi \equiv 2(k+1)(k+2)(c_M)^k f_E$ is a technology index that combines the effects of better distribution of cost draws (lower c_M) and lower entry costs f_E . We assume that $c_M > \sqrt{[2(k+1)(k+2)\gamma f_E]/(L\Omega)}$ in order to ensure $c_D < c_M$ as was previously anticipated. We also note that, as the customization cost for non-core varieties becomes infinitely large ($\omega \rightarrow 0$), multi-product flexibility Ω goes to 1, and (14) then boils down to the single-product case studied by Melitz and Ottaviano (2008).

2.5 Equilibrium with Multi-Product Firms

Equation (14) summarizes how technology (referenced by the distribution of cost draws and the sunk entry cost), market size, product differentiation, and multi-product flexibility affect the toughness of competition in the market equilibrium. Increases in market size, technology improvements (a fall in c_M or f_E), and increases in product substitutability (a rise in γ) all lead to tougher competition in the market and thus to an equilibrium with a lower cost cutoff c_D . As multi-product flexibility Ω increases, firms respond by introducing more products. This additional production is skewed towards the better performing firms and also leads to tougher competition and a lower c_D cutoff.

A market with tougher competition (lower c_D) also features more product variety M and a lower average price \bar{p} (due to the combined effect of product selection towards lower cost varieties and of lower markups). Both of these contribute to higher welfare U . Given our Pareto parametrization, we can write all of these variables as simple closed form functions of the cost cutoff c_D :

$$M = \frac{2(k+1)\gamma}{\eta} \frac{\alpha - c_D}{c_D}, \quad \bar{p} = \frac{2k+1}{2k+2} c_D, \quad U = 1 + \frac{1}{2\eta} (\alpha - c_D) \left(\alpha - \frac{k+1}{k+2} c_D \right). \quad (15)$$

Increases in the toughness of competition do not affect the average number of varieties produced per firm $M/N = \Omega$ because the mass of surviving firms N rises by the same proportion as the mass of produced varieties M .¹⁰ However, each firm responds to tougher competition by dropping its worst performing varieties (highest m) and reducing the number of varieties produced $M(c)$.¹¹ The

¹⁰This exact offsetting effect between the number of firms and the number of products is driven by our functional form assumptions. However, the downward shift in $M(c)$ in response to competition (described next) holds for a much more general set of parametrizations.

¹¹To be precise, the number of produced varieties $M(c)$ weakly decreases: if the change in the cutoff c_D is small enough, then some firms may still produce the same number of varieties. For other firms with high cost c , $M(c)$ drops to zero which implies firm exit.

selection of firms with respect to exit explains how the average number of products produced per firm can remain constant: exiting firms are those with the highest cost c who produce the fewest number of products.

3 Competition, Product Mix, and Productivity

We now investigate the link between toughness of competition and productivity at both the firm and aggregate level. We just described how tougher competition affects the selection of both firms in a market, and of the products they produce: high cost firms exit, and firms drop their high cost products. These selection effects induce productivity improvements at both the firm and the aggregate level.¹²

However, our model features an important additional channel that links tougher competition to higher firm and aggregate productivity. This new channel operates through the effect of competition on a firm's product mix. Tougher competition induces multi-product firms to skew production towards their better performing varieties (closer to their core competency). Thus, holding a multi-product firm's product range fixed, an increase in competition leads to an increase in that firm's productivity. Aggregating across firms, this product mix response also generates an aggregate productivity gain from tougher competition, over and above the effects from firm and product selection.

We have not yet defined how firm and aggregate productivity are measured. We start with the aggregation of output, revenue, and cost (employment) at the firm level. For any firm c , this is simply the sum of output, revenue, and cost over all varieties produced:

$$Q(c) \equiv \sum_{m=0}^{M(c)-1} q(v(m, c)), \quad R(c) \equiv \sum_{m=0}^{M(c)-1} r(v(m, c)), \quad C(c) \equiv \sum_{m=0}^{M(c)-1} v(m, c) q(v(m, c)). \quad (16)$$

One measure of firm productivity is simply output per worker $\Phi(c) \equiv Q(c)/C(c)$. This productivity measure does not have a clear empirical counterpart for multi-product firms, as output units for each product are normalized so that one unit of each product generates the same utility for the consumer (this is the implicit normalization behind the product symmetry in the utility function). A firm's deflated sales per worker $\Phi_R(c) \equiv [R(c)/\bar{P}]/C(c)$ provides another productivity measure that has a clear empirical counterpart. For this productivity measure, we need to define the price

¹²This effect of product scope on firm productivity is emphasized by Bernard et al (forthcoming) and Eckel and Neary (2010).

deflator \bar{P} . We choose

$$\bar{P} \equiv \frac{\int_0^{c_D} R(c)dG(c)}{\int_0^{c_D} Q(c)dG(c)} = \frac{k+1}{k+2}c_D.$$

This is the average of all the variety prices $p(v)$ weighted by their output share. We could also have used the unweighted price average \bar{p} that we previously defined, or an average weighted by a variety's revenue share (i.e. its market share) instead of output share. In our model, all of these price averages only differ by a multiplicative constant, so the effects of competition (changes in the cutoff c_D) on productivity will not depend on this choice of price averages.¹³ We define the aggregate counterparts to our two firm productivity measures as industry output per worker and industry deflated sales per worker:

$$\bar{\Phi} \equiv \frac{\int_0^{c_D} Q(c)dG(c)}{\int_0^{c_D} C(c)dG(c)}, \quad \bar{\Phi}_R = \frac{[\int_0^{c_D} R(c)dG(c)]/\bar{P}}{\int_0^{c_D} C(c)dG(c)}.$$

Our choice of the price deflator \bar{P} then implies that these two aggregate productivity measures coincide:¹⁴

$$\bar{\Phi} = \bar{\Phi}_R = \frac{k+2}{k} \frac{1}{c_D}. \quad (17)$$

Equation (17) summarizes the overall effect of tougher competition on aggregate productivity gains. This aggregate response of productivity combines the effects of competition on both firm productivity and inter-firm reallocations (including entry and exit). We now detail how tougher competition induces improvements in firm productivity through its impact on a firm's product mix. In the appendix, we show that both firm productivity measures, $\Phi(c)$ and $\Phi_R(c)$, increase for all multi-product firms when competition increases (c_D decreases). The key component of this proof is that, holding a firm's product scope constant (a given number $M > 1$ of non-core varieties produced), firm productivity over that product scope (output or deflated sales of those M products per worker producing those products) increases whenever competition increases. This effect of competition on firm productivity, by construction, is entirely driven by the response of the firm's product mix.

To isolate this product mix response to competition, consider two varieties m and m' produced by a firm with cost c . Assume that $m < m'$ so that variety m is closer to the core. The ratio of

¹³As we previously reported, the unweighted price average is $\bar{p} = [(2k+1)/(2k+2)]c_D$; and the average weighted by market share is $[(6k+2k^2+3)/(2k^2+8k+6)]c_D$.

¹⁴If we had picked one of the other price averages, the two aggregate productivity measures would differ by a multiplicative constant.

the firm's output of the two varieties is given by

$$\frac{q(v(m, c))}{q(v(m', c))} = \frac{c_D - \omega^{-m}c}{c_D - \omega^{-m'}c}.$$

As competition increases (c_D decreases), this ratio increases, implying that the firm skews its production towards its core varieties. This happens because the increased competition increases the price elasticity of demand for all products. At a constant relative price $p(v(m, c))/p(v(m', c))$, the higher price elasticity translates into higher relative demand $q(v(m, c))/q(v(m', c))$ and sales $r(v(m, c))/r(v(m', c))$ for good m (relative to m').¹⁵ In our specific demand parametrization, there is a further increase in relative demand and sales, because markups drop more for good m than m' , which implies that the relative price $p(v(m, c))/p(v(m', c))$ decreases.¹⁶ It is this reallocation of output towards better performing products (also mirrored by a reallocation of production labor towards those products) that generates the productivity increases within the firm. In other words, tougher competition skews the distribution of employment, output, and sales towards the better performing varieties (closer to the core), while it flattens the firm's distribution of prices.

In the open economy version of our model that we develop in the next section, we show how firms respond to tougher competition in export markets in very similar ways by skewing their exported product mix towards their better performing products. Our empirical results confirm a strong effect of such a link between competition and product mix.

4 Open Economy

We now turn to the open economy in order to examine how market size and geography determine differences in the toughness of competition across markets – and how the latter translates into differences in the exporters' product mix. We allow for an arbitrary number of countries and asymmetric trade costs. Let J denote the number of countries, indexed by $l = 1, \dots, J$. The markets are segmented, although any produced variety can be exported from country l to country h subject to an iceberg trade cost $\tau_{lh} > 1$. Thus, the delivered cost for variety m exported to country h by a firm with core competency c in country l is $\tau_{lh}v(m, c) = \tau_{lh}\omega^{-m}c$.

¹⁵For the result on relative sales, we are assuming that demand is elastic.

¹⁶Good m closer to the core initially has a higher markup than good m' ; see (7).

4.1 Equilibrium with Asymmetric Countries

Let p_l^{\max} denote the price threshold for positive demand in market l . Then (4) implies

$$p_l^{\max} = \frac{1}{\eta M_l + \gamma} (\gamma \alpha + \eta M_l \bar{p}_l), \quad (18)$$

where M_l is the total number of products selling in country l (the total number of domestic and exported varieties) and \bar{p}_l is their average price. Let $\pi_{ll}(v)$ and $\pi_{lh}(v)$ represent the maximized value of profits from domestic and export sales to country h for a variety with cost v produced in country l . (We use the subscript ll to denote domestic sales: by firms in l to destination l .) The cost cutoffs for profitable domestic production and for profitable exports must satisfy:

$$\begin{aligned} v_{ll} &= \sup \{c : \pi_{ll}(v) > 0\} = p_l^{\max}, \\ v_{lh} &= \sup \{c : \pi_{lh}(v) > 0\} = \frac{p_h^{\max}}{\tau_{lh}}, \end{aligned} \quad (19)$$

and thus $v_{lh} = v_{hh}/\tau_{lh}$. As was the case in the closed economy, the cutoff v_{ll} , $l = 1, \dots, J$, summarizes all the effects of market conditions in country l relevant for all firm performance measures. The profit functions can then be written as a function of these cutoffs:

$$\begin{aligned} \pi_{ll}(v) &= \frac{L_l}{4\gamma} (v_{ll} - v)^2, \\ \pi_{lh}(v) &= \frac{L_h}{4\gamma} \tau_{lh}^2 (v_{lh} - v)^2 = \frac{L_h}{4\gamma} (v_{hh} - \tau_{lh}v)^2. \end{aligned} \quad (20)$$

As in the closed economy, $c_{ll} = v_{ll}$ will be the cutoff for firm survival in country l (cutoff for sales to domestic market l). Similarly, $c_{lh} = v_{lh}$ will be the firm export cutoff (no firm with $c > c_{lh}$ can profitably export any varieties from l to h). A firm with core competency c will produce all varieties m such that $\pi_{ll}(v(m, c)) \geq 0$; it will export to h the subset of varieties m such that $\pi_{lh}(v(m, c)) \geq 0$. The total number of varieties produced and exported to h by a firm with cost c in country l are thus

$$\begin{aligned} M_{ll}(c) &= \begin{cases} 0 & \text{if } c > c_{ll}, \\ \max \{m \mid c \leq \omega^m c_{ll}\} + 1 & \text{if } c \leq c_{ll}, \end{cases} \\ M_{lh}(c) &= \begin{cases} 0 & \text{if } c > c_{lh}, \\ \max \{m \mid c \leq \omega^m c_{lh}\} + 1 & \text{if } c \leq c_{lh}. \end{cases} \end{aligned}$$

We can then define a firm's total domestic and export profits by aggregating over these varieties:

$$\Pi_{ll}(c) = \sum_{m=0}^{M_{ll}(c)-1} \pi_{ll}(v(m, c)), \quad \Pi_{lh}(c) = \sum_{m=0}^{M_{lh}(c)-1} \pi_{lh}(v(m, c)).$$

Entry is unrestricted in all countries. Firms choose a production location prior to entry and paying the sunk entry cost. We assume that the entry cost f_E and cost distribution $G(c)$ are common across countries (although this can be relaxed).¹⁷ We maintain our Pareto parametrization (12) for this distribution. A prospective entrant's expected profits will then be given by

$$\begin{aligned} & \int_0^{c_{ll}} \Pi_{ll}(c) dG(c) + \sum_{h \neq l} \int_0^{c_{lh}} \Pi_{lh}(c) dG(c) \\ &= \sum_{m=0}^{\infty} \left[\int_0^{\omega^m c_{ll}} \pi_{ll}(\omega^{-m} c) dG(c) \right] + \sum_{h \neq l} \sum_{m=0}^{\infty} \left[\int_0^{\omega^m c_{lh}} \pi_{lh}(\omega^{-m} c) dG(c) \right] \\ &= \frac{1}{2\gamma(k+1)(k+2)c_M^k} \left[L_l \Omega c_{ll}^{k+2} + \sum_{h \neq l} L_h \Omega \tau_{lh}^2 c_{lh}^{k+2} \right] \\ &= \frac{\Omega}{2\gamma(k+1)(k+2)c_M^k} \left[L_l c_{ll}^{k+2} + \sum_{h \neq l} L_h \tau_{lh}^{-k} c_{hh}^{k+2} \right]. \end{aligned}$$

Setting the expected profit equal to the entry cost yields the free entry conditions:

$$\sum_{h=1}^J \rho_{lh} L_h c_{hh}^{k+2} = \frac{\gamma\phi}{\Omega} \quad l = 1, \dots, J. \quad (21)$$

where $\rho_{lh} \equiv \tau_{lh}^{-k} < 1$ is a measure of 'freeness' of trade from country l to country h that varies inversely with the trade costs τ_{lh} . The technology index ϕ is the same as in the closed economy case.

The free entry conditions (21) yield a system of J equations that can be solved for the J equilibrium domestic cutoffs using Cramer's rule:

$$c_{hh} = \left(\frac{\gamma\phi}{\Omega} \frac{\sum_{l=1}^J |C_{lh}|}{|\mathcal{P}|} \frac{1}{L_h} \right)^{\frac{1}{k+2}}, \quad (22)$$

¹⁷Differences in the support for this distribution could also be introduced as in Melitz and Ottaviano (2008).

where $|\mathcal{P}|$ is the determinant of the trade freeness matrix

$$\mathcal{P} \equiv \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1M} \\ \rho_{21} & 1 & \cdots & \rho_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{M1} & \rho_{M2} & \cdots & 1 \end{pmatrix},$$

and $|C_{lh}|$ is the cofactor of its ρ_{lh} element. Cross-country differences in cutoffs now arise from two sources: own country size (L_h) and geographical remoteness, captured by $\sum_{l=1}^J |C_{lh}| / |\mathcal{P}|$. Central countries benefiting from a large local market have lower cutoffs, and exhibit tougher competition, than peripheral countries with a small local market.

As in the closed economy, the threshold price condition in country h (18), along with the resulting Pareto distribution of all prices for varieties sold in h (domestic prices and export prices have an identical distribution in country h) yield a zero-cutoff profit condition linking the variety cutoff $v_{hh} = c_{hh}$ to the mass of varieties sold in country h :

$$M_h = \frac{2(k+1)\gamma}{\eta} \frac{\alpha - c_{hh}}{c_{hh}}. \quad (23)$$

Given a positive mass of entrants $N_{E,l}$ in country l , there will be $G(c_{lh})N_{E,l}$ firms exporting $\Omega\rho_{lh}G(c_{lh})N_{E,l}$ varieties to country h . Summing over all these varieties (including those produced and sold in h) yields¹⁸

$$\sum_{l=1}^J \rho_{lh} N_{E,l} = \frac{M_h}{\Omega c_{hh}^k}.$$

The latter provides a system of J linear equations that can be solved for the number of entrants in the J countries using Cramer's rule:¹⁹

$$N_{E,l} = \frac{\phi\gamma}{\Omega\eta(k+2)f_E} \sum_{h=1}^J \frac{(\alpha - c_{hh}) |C_{lh}|}{c_{hh}^{k+1} |\mathcal{P}|}. \quad (24)$$

As in the closed economy, the cutoff level completely summarizes the distribution of prices as well as all the other performance measures. Hence, the cutoff in each country also uniquely determines welfare in that country. The relationship between welfare and the cutoff is the same as in the closed economy (see (15)).

¹⁸Recall that $c_{hh} = \tau_{lh} c_{lh}$.

¹⁹We use the properties that relate the freeness matrix P and its transpose in terms of determinants and cofactors.

4.2 Bilateral Trade Patterns with Firm and Product Selection

We have now completely characterized the multi-country open economy equilibrium. Selection operates at many different margins: a subset of firms survive in each country, and a smaller subset of those export to any given destination. Within a firm, there is an endogenous selection of its product range (the range of product produced); those products are all sold on the firm’s domestic market, but only a subset of those products are sold in each export market. In order to keep our multi-country open economy model as tractable as possible, we have assumed a single bilateral trade cost τ_{lh} that does not vary across firms or products. This simplification implies some predictions regarding the ordering of the selection process across countries and products that is overly rigid. Since τ_{lh} does not vary across firms in l contemplating exports to h , then all those firms would face the same ranking of export market destinations based on the toughness of competition in that market, c_{hh} , and the trade cost to that market τ_{lh} . All exporters would then export to the country with the highest c_{hh}/τ_{lh} , and then move down the country destination list in decreasing order of this ratio until exports to the next destination were no longer profitable. This generates a “pecking order” of export destinations for exporters from a given country l . Eaton, Kortum, and Kramarz (forthcoming) show that there is such a stable ranking of export destinations for French exporters. Needless to say, the empirical prediction for the ordered set of export destinations is not strictly adhered to by every French exporter (some export to a given destination without also exporting to all the other higher ranked destinations). Eaton, Kortum, and Kramarz formally show how some idiosyncratic noise in the bilateral trading cost can explain those departures from the dominant ranking of export destinations. They also show that the empirical regularities for the ranking of export destinations are so strong that one can easily reject the notion of independent export destination choices by firms.

Our model features a similar rigid ordering within a firm regarding the products exported across destinations. Without any variation in the bilateral trade cost τ_{lh} across products, an exporter from l would always exactly follow its domestic core competency ladder when determining the range of products exported across destinations: an exporter would never export variety $m' > m$ unless it also exported variety m to any given destination. Just as we described for the prediction of country rankings, we clearly do not expect the empirical prediction for product rankings to hold exactly for all firms. Nevertheless, a similar empirical pattern emerges highlighting a stable ranking of products for each exporter across export destinations.²⁰ We empirically describe the substantial

²⁰Bernard et al (forthcoming) and Arkolakis and Muendler (2010) report that there is such a stable ordering of a

extent of this ranking stability for French exporters in our next section.

Putting together all the different margins of trade, we can use our model to generate predictions for aggregate bilateral trade. An exporter in country l with core competency c generates export sales of variety m to country h equal to (assuming that this variety is exported):

$$r_{lh}(v(m, c)) = \frac{L_h}{4\gamma} \left[v_{hh}^2 - (\tau_{lh}v(m, c))^2 \right]. \quad (25)$$

Aggregate bilateral trade from l to h is then:

$$\begin{aligned} \text{EXP}_{lh} &= N_{E,l} \Omega \rho^{lh} \int_0^{c_{lh}} r_{lh}(v(m, c)) dG(v) \\ &= \frac{\Omega}{2\gamma(k+2)c_M^k} \times N_{E,l} \times c_{hh}^{k+2} L_h \times \rho_{lh}. \end{aligned} \quad (26)$$

Thus, aggregate bilateral trade follows a standard gravity specification based on country fixed effects (separate fixed effects for the exporter and importer) and a bilateral term that captures the effects of all bilateral barriers/enhancers to trade.²¹

5 Exporters' Product Mix Across Destinations

We previously described how, in the closed economy, firms respond to increases in competition in their market by skewing their product mix towards their core products. We also analyzed how this product mix response generated increases in firm productivity. We now show how differences in competition across export market destinations induce exporters to those markets to respond in very similar ways: when exporting to markets with tougher competition, exporters skew their product level exports towards their core products. We proceed in a similar way as we did for the closed economy by examining a given firm's ratio of exports of two products m' and m , where m is closer to the core. In anticipation of our empirical work, we write the ratio of export sales (revenue not output), but the ratio of export quantities responds to competition in identical ways. Using (25), we can write this sales ratio:

$$\frac{r_{lh}(v(m, c))}{r_{lh}(v(m', c))} = \frac{c_{hh}^2 - (\tau_{lh}\omega^{-m}c)^2}{c_{hh}^2 - (\tau_{lh}\omega^{-m'}c)^2}. \quad (27)$$

firm's product line for U.S. and Brazilian firms.

²¹This type of structural gravity specification with country fixed-effects is generated by a large set of different modeling frameworks. See Feenstra (2004) for further discussion of this topic. In (26), we do not further substitute out the endogenous number of entrants and cost cutoff based on (22) and (24). This would lead to just a different functional form for the country fixed effects.

Tougher competition in an export market (lower c_{hh}) increases this ratio, which captures how firms skew their exports toward their core varieties (recall that $m' > m$ so variety m is closer to the core). The intuition behind this result is very similar to the one we described for the closed economy. Tougher competition in a market increases the price elasticity of demand for all goods exported to that market. As in the closed economy, this skews relative demand and relative export sales towards the goods closer to the core. In our empirical work, we focus on measuring this effect of tougher competition across export market destinations on a firm's exported product mix.

We could also use (27) to make predictions regarding the impact of the bilateral trade cost τ_{lh} on a firm's exported product mix: Higher trade costs raise the firm's delivered cost and lead to a higher export ratio. The higher delivered cost increase the competition faced by an exporting firm, as it then competes against domestic firms that benefit from a greater cost advantage. However, this comparative static is very sensitive to the specification for the trade cost across a firm's product ladder. If trade barriers induce disproportionately higher trade costs on products further away from the core, then the direction of this comparative static would be reversed. Furthermore, identifying the independent effect of trade barriers on the exporters' product mix would also require micro-level data for exporters located in many different countries (to generate variation across both origin and destination of export sales). Our data 'only' covers the export patterns for French exporters, and does not give us this variation in origin country. For these reasons, we do not emphasize the effect of trade barriers on the product mix of exporters. In our empirical work, we will only seek to control for a potential correlation between bilateral trade barriers with respect to France and the level of competition in destination countries served by French exporters.

As was the case for the closed economy, the skewing of a firm's product mix towards core varieties also entails increases in firm productivity. Empirically, we cannot separately measure a firm's productivity with respect to its production for each export market. However, we can theoretically define such a productivity measure in an analogous way to $\Phi(c) \equiv Q(c)/C(c)$ for the closed economy. We thus define the productivity of firm c in l for its exports to destination h as $\Phi_{lh}(c) \equiv Q_{lh}(c)/C_{lh}(c)$, where $Q_{lh}(c)$ are the total units of output that firm c exports to h , and $C_{lh}(c)$ are the total labor costs incurred by firm c to produce those units.²² In the appendix,

²²In order for this productivity measure to aggregate up to overall country productivity, we incorporate the productivity of the transportation/trade cost sector into this productivity measure. This implies that firm c employs the labor units that are used to produce the "melted" units of output that cover the trade cost; Those labor units are thus included in $C_{lh}(c)$. The output of firm c is measured as valued-added, which implies that those "melted" units are not included in $Q_{lh}(c)$ (the latter are the number of units produced by firm c that are consumed in h). Separating out the productivity of the transportation sector would not affect our main comparative static with respect to toughness of competition in the export market.

we show that this export market-specific productivity measure (as well as the associated measure $\Phi_{R,lh}(c)$ based on deflated sales) increases with the toughness of competition in that export market. In other words, $\Phi_{lh}(c)$ and $\Phi_{R,lh}(c)$ both increase when c_{hh} decreases. Thus, changes in exported product mix also have important repercussions for firm productivity.

6 Empirical Analysis

6.1 Skewness of Exported Product Mix

We now test the main prediction of our model regarding the impact of competition across export market destinations on a firm's exported product mix. Our model predicts that tougher competition in an export market will induce firms to lower markups on all their exported products and therefore skew their export sales towards their best performing products. We thus need data on a firm's exports across products and destinations. We use comprehensive firm-level data on annual shipments by all French exporters to all countries in the world for a set of more than 10,000 goods. Firm-level exports are collected by French customs and include export sales for each 8-digit (combined nomenclature) product by destination country.²³ A firm located in the French metropolitan territory in 2003 (the year we use) must report this detailed export information so long as the following criteria are met: For within EU exports, the firm's annual trade value exceeds 100,000 Euros;²⁴ and for exports outside the EU, the exported value to a destination exceeds 1,000 Euros or a weight of a ton. Despite these limitations, the database is nearly comprehensive. In 2003, 100,033 firms report exports across 229 destination countries (or territories) for 10,072 products. This represents data on over 2 million shipments. We restrict our analysis to export data in manufacturing industries, mostly eliminating firms in the service and wholesale/distribution sector to ensure that firms take part in the production of the goods they export.²⁵ This leaves us with data on over a million shipments by firms in the whole range of manufacturing sectors. We also drop observations for firms that the French national statistical institute reports as having an affiliate abroad. This avoids the issue that multinational firms may substitute exports of some of their best performing products with affiliate production in the destination country (following the export versus FDI trade-off described in Helpman et al (2004)). We therefore limit our analysis to firms that do not have this possibility, in order to reduce noise in the product export rankings.

²³We thank the French customs administration for making this data available to researchers at CEPII.

²⁴If that threshold is not met, firms can choose to report under a simplified scheme without supplying export destinations. However, in practice, many firms under that threshold report the detailed export destination information.

²⁵Some large distributors such as Carrefour account for a disproportionate number of annual shipments.

In order to measure the skewness of a firm’s exported product mix across destinations, we first need to make some assumptions regarding the empirical measurement of a firm’s product ladder. We start with the most direct counterpart to our theoretical model, which assumes that the firm’s product ladder does *not* vary across destinations. For this measure, we rank all the products exported by a firm according to the value of exports to the world, and use this ranking as an indicator for the product rank m .²⁶ We call this the firm’s *global* product rank. An alternative is to measure a firm’s product rank for each destination based on the firm’s exports sales to that destination. We call this the firm’s *local* product rank. Empirically, this local product ranking can vary across destinations. However, as we alluded to earlier, this local product ranking is remarkably stable across destinations.

The Spearman rank correlation between a firm’s local and global rankings (in each export market destination) is .68.²⁷ Naturally, this correlation might be partly driven by firms that export only one product to one market, for which the global rank has to be equal to the local rank. In Table 1, we therefore report the rank correlation as we gradually restrict the sample to firms that export many products to many markets. The bottom line is that this correlation remains quite stable: for firms exporting more than 50 products to more than 50 destinations, the correlation is still .58. Another possibility is that this correlation is different across destination income levels. Restricting the sample to the top 50 or 20% richest importers hardly changes this correlation (.69 and .71 respectively).²⁸

Table 1: Spearman Correlations Between Global and Local Rankings

Firms exporting at least: to # countries	# products				
	1	2	5	10	50
1	67.93%	67.78%	67.27%	66.26%	59.39%
2	67.82%	67.74%	67.28%	66.28%	59.39%
5	67.55%	67.51%	67.2%	66.3%	59.43%
10	67.02%	67%	66.82%	66.12%	59.46%
50	61.66%	61.66%	61.64%	61.53%	58.05%

Although high, this correlation still highlights substantial departures from a steady global product ladder. A natural alternative is therefore to use the local product rank when measuring the

²⁶We experimented ranking products for each firm based on the number of export destinations; and obtained very similar results to the ranking based on global export sales.

²⁷Arkolakis and Muendler (2010) also report a huge amount of stability in the local rankings across destinations. The Spearman rank coefficient they report is .837. Iacovone and Javorcik (2008) report a rank correlation of .76 between home and export sales of Mexican firms.

²⁸We nevertheless separately report our regression results for those restricted sample of countries based on income.

skewness of a firm’s exported product mix. In this interpretation, the identity of the core (or other rank number) product can change across destinations. We thus use both the firm’s global and local product rank to construct the firm’s destination-specific export sales ratio $r_{lh}(v(m, c))/r_{lh}(v(m', c))$ for $m < m'$. Since many firms export few products to many destinations, increasing the higher product rank m' disproportionately reduces the number of available firm/destination observations. For most of our analysis, we pick $m = 0$ (core product) and $m' = 1$, but also report results for $m' = 2$.²⁹ Thus, we construct the ratio of a firm’s export sales to every destination for its best performing product (either globally, or in each destination) relative to its next best performing product (again, either globally, or in each destination). The local ratios can be computed so long as a firm exports at least two products to a destination (or three when $m' = 2$). The global ratios can be computed so long as a firm exports its top (in terms of world exports) two products to a destination. We thus obtain these measures that are firm c and destination h specific, so long as those criteria are met (there is no variation in origin $l = \text{France}$). We use those ratios in logs, so that they represent percentage differences in export sales. We refer to the ratios as either local or global, based on the ranking method used to compute them. Lastly, we also constrain the sample so that the two products considered belong to the same 2-digit product category (there are 97 such categories). This eliminates ratios based on products that are in completely different sectors; however, this restriction hardly impacts our reported results.

We construct a third measure that seeks to capture changes in skewness of a firm’s exported product mix over the entire range of exported products (instead of being confined to the top two or three products). We use several different skewness statistics for the distribution of firm export sales to a destination: the standard deviation of log export sales, a Herfindhal index, and a Theil index (a measure of entropy). Since these statistics are independent of the identity of the products exported to a destination, they are “local” by nature, and do not have any global ranking counterpart. These statistics can be computed for every firm-destination combination where the firm exports two or more products. The Theil and standard deviation statistics have the attractive property that they are invariant to truncation from below when the underlying distribution is Pareto; this distribution provides a very good fit for the within-firm distribution of export sales to a destination.

We graphically show the fit to the Pareto distribution in Figure 2. We plot the average share of a firm’s export sales by product against that product’s local rank.³⁰ We restrict the sample to the

²⁹We also obtain very similar results for $m = 1$ and $m' = 2$.

³⁰Bernard et al (forthcoming) report a similar graph for U.S. firms exporting 10 products to Canada. They also find a strong goodness of fit to the Pareto distribution.

top 50 products exported by firms that export between 50 and 100 products. A Pareto distribution for within-firm export sales implies a straight line on the log-log figure scale. Although there are clearly departures from Pareto at both ends of the distribution, the tightness of the relationship is quite striking. We also investigate the goodness of fit to the Pareto distribution by running *within firm-destination* regressions of log rank on log exports (for the 7570 French firms exporting more than 10 products and less than 50 in our sample). The median R-Squared is .906, indicating a very good fit of the Pareto distribution for export sales at the firm-destination level. Thus, the truncation of export sales should not bias our dispersion measured based on the Theil and standard deviation statistics.

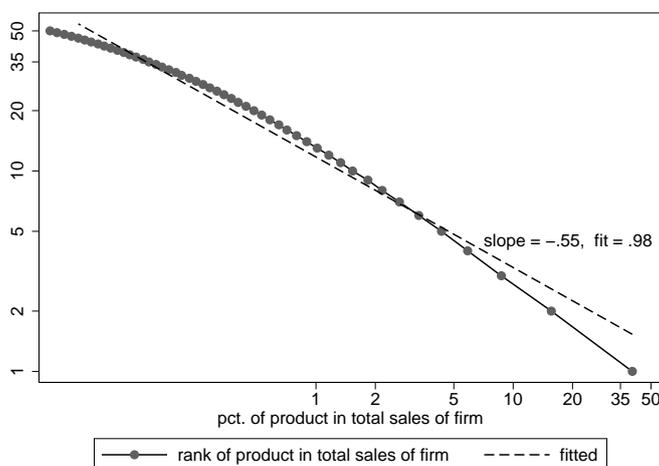


Figure 2: Average share of product sales depending on the rank of the product.

6.2 Toughness of Competition Across Destinations and Bilateral Controls

Our theoretical model predicts that the toughness of competition in a destination is determined by that destination's size, and by its geography (proximity to other big countries). We control for country size using GDP expressed in a common currency at market exchange rates. We now seek a control for the geography of a destination that does not rely on country-level data for that destination. We use the *supply potential* concept introduced by Redding and Venables (2004) as such a control. In words, the supply potential is the aggregate predicted exports to a destination based on a bilateral trade gravity equation (in logs) with both exporter and importer fixed effects and the standard bilateral measures of trade barriers/enhancers. We construct a related measure of a destination's *foreign* supply potential that does not use the importer's fixed effect when predicting

aggregate exports to that destination. By construction, foreign supply potential is thus uncorrelated with the importer’s fixed-effect. It is closely related to the construction of a country’s market potential (which seeks to capture a measure of predicted import demand for a country).³¹ The construction of the supply potential measures is discussed in greater detail in Redding and Venables (2004); we use the foreign supply measure for the year 2003 from Head and Mayer (2011) who extend the analysis to many more countries and more years of data.³² Since we only work with the foreign supply potential measure, we drop the qualifier ‘foreign’ when we subsequently refer to this variable.

We also use a set of controls for bilateral trade barriers/enhancers (τ in our model) between France and the destination country: distance, contiguity, colonial links, common-language, and dummies for membership of Regional Trading Agreements, GATT/WTO, and a common currency area (the eurozone in this case).³³

6.3 Results

Before reporting the regression results of the skewness measures on the destination country measures, we first show some scatter plots for the global ratio against both destination country GDP and our measure of supply potential. These are displayed in Figures 3 and 4. For each destination, we use the mean global ratio across exporting firms. Since the firm-level measure is very noisy, the precision of the mean increases with the number of available firm data points (for each destination). We first show the scatter plots using all available destinations, with symbol weights proportional to the number of available firm observations, and then again dropping any destination with fewer than 250 exporting firms.³⁴ Those scatter plots show a very strong positive correlation between the export share ratios and the measures of toughness of competition in the destination. Absent any variation in the toughness of competition across destinations – such as in a world with monopolistic competition and C.E.S. preferences where markups are exogenously fixed – the variation in the relative export shares should be white noise. The data clearly show that variations in competition (at least as proxied by country size and supplier potential) are strong enough to induce large variations

³¹Redding and Venables (2004) show that this construction for supply potential (and the similar one for market potential) is also consistent with its theoretical counterpart in a Dixit-Stiglitz-Krugman model. They construct those measures for a cross-section of 100 countries in 1994. Head and Mayer (2011) use the same methodology to cover more countries and a longer time period.

³²As is the case with market potential, a country’s supplier potential is strongly correlated with that country’s GDP: big trading economies tend to be located near one-another. The supply potential data is available online at <http://www.cepii.fr/anglaisgraph/bdd/marketpotentials.htm>

³³All those variables are available at <http://www.cepii.fr/anglaisgraph/bdd/gravity.htm>

³⁴Increasing that threshold level for the number of exporters slightly increases the fit and slope of the regression line through the scatter plot.

to the center of the biggest regional trading group in the world. Thus, distance from France is highly correlated with “good” geography and hence a high supply potential for that destination: the correlation between log distance and log supply potential is 78%. Therefore, when we introduce all the controls for bilateral trade barriers to our specification, it is not surprising that there is too much co-linearity with the destination’s supply potential to separately identify the independent effect of the latter.³⁶ These results are reported in columns (2) and (5) of Table 2. Although the coefficient for supply potential is no longer significant due to this co-linearity problem, the effect of country size on the skewness of export sales remain highly significant. Other than country size, the only other variable that is significant (at 5% or below) is the effect of a common currency: export sales to countries in the Eurozone display vastly higher skewness. However, we must exercise caution when interpreting this effect. Due to the lack of variation in origin country, we cannot say whether this captures the effect of a common currency between the destination and France, or whether this is an independent effect of the Euro.³⁷

Although we do not have firm-product-destination data for countries other than France, bilateral aggregate data is available for the full matrix of origins-destinations in the world. Our theoretical model predicts a bilateral gravity relationship (26) that can be exploited to recover the combined effect of bilateral trade barriers as a single parameter (τ_{lh} in our model). The only property of our gravity relationship that we exploit is that bilateral trade can be decomposed into exporter and importer fixed effects, and a bilateral component that captures the joint effect of trade barriers.³⁸ We use the same bilateral gravity specification that we previously used to construct supply potential (again, in logs). We purge bilateral flows from both origin and destination fixed effects, to keep only the contribution of bilateral barriers to trade. This gives us an estimate for the bilateral log freeness of trade between all country pairs ($\ln \rho_{lh}$).³⁹ We use the subset of this predicted data where France is the exporting country. Looking across destinations, this freeness of trade variable is still highly correlated with distance from France (the correlation with log distance is 60%); but it is substantially less correlated with the destination’s supply potential than distance from France (the

³⁶As we mentioned, distance by itself introduces a huge amount of co-linearity with supply potential. The other bilateral trade controls then further exacerbate this problem (membership in the EU is also strongly correlated with “good” geography and hence supply potential).

³⁷If this is a destination Euro effect, then this would fit well with our theoretical prediction for the effect of tougher competition in Euro markets on the skewness of export sales.

³⁸This property of gravity equations is not specific to our model. It can be generated by a very large class of models. Arkolakis et al. (forthcoming) and Head and Mayer (2011) discuss all the different models that lead to a similar gravity decomposition.

³⁹Again, we emphasize that there is a very large class of models that would generate the same procedure for recovering bilateral freeness of trade.

correlation between freeness of trade and log supply potential is 40%, much lower than the 78% correlation between log distance and log supply potential). This greatly alleviates the co-linearity problem while allowing us to control for the relevant variation induced by bilateral trade barriers (i.e. calculated based upon their impact on bilateral trade flows).

Columns (3) and (6) of Table 2 report the results using this constructed freeness of trade measure as our control for the independent effect of bilateral trade barriers on export skewness. The results are very similar to our initial ones without any bilateral controls: country size and supply potential both have a strong and highly significant effect on the skewness of export sales. These effects are also economically significant. The coefficient on country size can be directly interpreted as an elasticity for the sales ratio with respect to country GDP. The 0.107 elasticity for the global ratio implies that an increase in destination GDP from that of the Czech Republic to German GDP (an increase from the 79th to 99th percentile in the world's GDP distribution in 2003) would induce French firms to increase their relative exports of their best product (relative to their next best global product) by 42.1%: from an observed mean ratio of 20 in 2003 to 28.4.

We now investigate the robustness of this result to different skewness measures, to the sample of destination countries, and to an additional control for destination GDP per capita. From here on out, we use our constructed freeness of trade measure to control for bilateral trade barriers.

We report the same set of results for the global sales ratio in Table 3 and for the local ratio in Table 4. The first column reproduces baseline estimation reported in columns (3) and (6) with the freeness of trade control. In column (2), we use the sales ratio of the best to third best product as our dependent skewness variable.⁴⁰ We then return to sales ratio based on best to next best for the remaining columns. In order to show that our results are not driven by unmeasured quality differences between the products shipped to developed and developing countries, we progressively restrict our sample of country destinations to a subset of richer countries. In column (3) we restrict destinations to those above the median country income, and in column (4), we only keep the top 20% of countries ranked by income (GDP per capita).⁴¹ In the fifth and last column, we keep the full sample of country destinations and add destination GDP per capita as a regressor in order to directly control for differences in preferences across countries (outside the scope of our theoretical model) tied to product quality and consumer income.⁴² All of these different specifications in Tables

⁴⁰We also experimented with the ratio for the second best to third best product, and obtained very similar results.

⁴¹Since French firms ship disproportionately more goods to countries with higher incomes, the number of observations drops very slowly with the number of excluded country destinations.

⁴²In particular, we want to allow consumer income to bias consumption towards higher quality varieties. If within-firm product quality is negatively related to its distance from the core product, then this would induce a positive

Table 2: Global and local export sales ratio: core ($m = 0$) product to second best ($m' = 1$) product

Dep. Var.	(1)	(2)	(3)	(4)	(5)	(6)
	Ratio of core to second product sales' regressions					
	Global ratio			Local ratio		
ln GDP	0.092*** (0.013)	0.083*** (0.012)	0.107*** (0.010)	0.073*** (0.008)	0.057*** (0.005)	0.077*** (0.006)
ln supply potential	0.067*** (0.016)	-0.017 (0.024)	0.044*** (0.014)	0.080*** (0.016)	0.018 (0.016)	0.068*** (0.013)
ln distance		-0.063 (0.043)			-0.046* (0.023)	
contiguity		0.013 (0.051)			-0.108 (0.081)	
colonial link		-0.060 (0.051)			-0.041 (0.043)	
common language		0.023 (0.050)			-0.048 (0.038)	
RTA		0.066 (0.059)			0.004 (0.033)	
common currency		0.182*** (0.047)			0.335*** (0.037)	
both in GATT		0.006 (0.046)			-0.033 (0.026)	
ln freeness of trade			0.096*** (0.026)			0.028 (0.017)
Constant	-0.000 (0.016)	0.000 (0.012)	-0.000 (0.014)	0.003 (0.012)	0.002 (0.011)	0.002 (0.013)
Observations	56096	56096	56092	96889	96889	96876
Within R ²	0.004	0.006	0.005	0.007	0.011	0.007

Note: All columns use Wooldridge's (2006) procedure: country-specific random effects on firm-demeaned data, with a robust covariance matrix estimation. Standard errors in parentheses. Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

3 and 4 confirm the robustness of our baseline results regarding the strong impact of both country size and geography on the firms' export ratios.⁴³

Table 3: Global export sales ratio: core product ($m = 0$) to product m'

	(1)	(2)	(3)	(4)	(5)
ln GDP	0.107*** (0.010)	0.155*** (0.031)	0.110*** (0.011)	0.096*** (0.012)	0.098*** (0.011)
ln supply potential	0.044*** (0.014)	0.111*** (0.033)	0.038*** (0.014)	0.022* (0.012)	0.036** (0.016)
ln freeness of trade	0.096*** (0.026)	0.020 (0.057)	0.113*** (0.032)	0.137*** (0.038)	0.092*** (0.026)
ln GDP per cap					0.025 (0.018)
$m' =$	1	2	1	1	1
Destination GDP/cap	all	all	top 50%	top 20%	all
Observations	56092	5688	50622	40963	56092
Within R ²	0.005	0.018	0.004	0.002	0.005

Note: All columns use Wooldridge's (2006) procedure: country-specific random effects on firm-demeaned data, with a robust covariance matrix estimation. Standard errors in parentheses. Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Lastly, we show that this effect of country size and geography on export skewness is not limited to the top 2-3 products exported by a firm to a destination. We now use our different statistics that measure the skewness of a firm's export sales over the entire range of exported products. The first three columns of Table 5 use the standard deviation, Herfindahl index, and Theil index for the distribution of the firm's export sales to each destination with our baseline specification (freeness of trade control for bilateral trade barriers and the full sample of destination countries). In the last three columns, we stick with the Theil index and report the same robustness specifications as we reported for the local and global sales ratio: We reduce the sample of destinations by country income, and add GDP per capita as an independent control with the full sample of countries. Throughout Table 5, we add a cubic polynomial in the number of exported products by the firm to the destination (those coefficients are not reported). This controls for any mechanical effect of the correlation between consumer income and the within-firm skewness of expenditure shares. This is the sign of the coefficient on GDP per capita that we obtain; that coefficient is statistically significant for the regressions based on the local product ranking.

⁴³When we restrict the sample of destinations to the top 20% of richest countries, then our co-linearity problem resurfaces between the supply potential and freeness of trade measures, and the coefficient on supply potential is no longer statistically significant at the 5% level (only at the 10% level).

Table 4: Local export sales ratio: core product ($m = 0$) to product m'

	(1)	(2)	(3)	(4)	(5)
ln GDP	0.077*** (0.006)	0.100*** (0.012)	0.083*** (0.011)	0.061*** (0.016)	0.066*** (0.008)
ln supply potential	0.068*** (0.013)	0.064*** (0.022)	0.051*** (0.018)	0.028* (0.016)	0.057*** (0.014)
ln freeness of trade	0.028 (0.017)	0.013 (0.042)	0.059 (0.039)	0.092* (0.052)	0.025 (0.017)
ln GDP per cap					0.029** (0.013)
$m' =$	1	2	1	1	1
Destination GDP/cap	all	all	top 50%	top 20%	all
Observations	96876	49554	84706	64652	96876
Within R ²	0.007	0.009	0.005	0.002	0.007

Note: All columns use Wooldridge's (2006) procedure: country-specific random effects on firm-demeaned data, with a robust covariance matrix estimation. Standard errors in parentheses. Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

number of exported products on the skewness statistic when the number of exported products is low. These results show how country size and geography increase the skewness of the firms' entire exported product mix. Using information on the entire distribution of exported sales increases the statistical precision of our estimates. The coefficients on country size and supply potential are significant well beyond the 1% threshold throughout all our different specifications.

7 Conclusion

In this paper, we have developed a model of multi-product firms that highlights how differences in market size and geography affect the within-firm distribution of export sales across destinations. This effect on the firms' product mix choice is driven by variations in the toughness of competition across markets. Tougher competition induces a downward shift in the distribution of markups across all products, and increases the relative market share of the better performing products. We test these predictions for a comprehensive set of French exporters, and find that market size and geography indeed have a very strong impact on their exported product mix across world destinations: French firms skew their export sales towards their better performing products in big destination markets, and markets where many exporters from around the world compete (high foreign supply potential markets). We take this as a strong indication that differences in the

Table 5: Skewness measures for export sales of all products

	(1)	(2)	(3)	(4)	(5)	(6)
ln GDP	0.141*** (0.010)	0.019*** (0.001)	0.047*** (0.002)	0.052*** (0.002)	0.047*** (0.003)	0.041*** (0.003)
ln supply potential	0.125*** (0.023)	0.016*** (0.002)	0.037*** (0.004)	0.033*** (0.004)	0.023*** (0.004)	0.031*** (0.004)
ln freeness of trade	0.096*** (0.036)	0.007* (0.004)	0.021** (0.009)	0.032** (0.013)	0.045** (0.022)	0.021** (0.009)
ln GDP per cap						0.013** (0.005)
Dep. Var.	s.d. ln x	herf	theil	theil	theil	theil
Destination GDP/cap	all	all	all	top 50%	top 20%	all
Observations	82090	82090	82090	73029	57076	82090
Within R ²	0.107	0.164	0.359	0.356	0.341	0.359

Note: All columns use Wooldridge's (2006) procedure: country-specific random effects on firm-demeaned data, with a robust covariance matrix estimation. Standard errors in parentheses. Significance levels: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. All columns include a cubic polynomial of the number of products exported by the firm to the country (also included in the within R²).

toughness of competition across export markets generate substantial responses in firm-level markups (indirectly revealed by the pronounced changes in the skewness of export sales). Trade models based on exogenous markups cannot explain this strong significant link between destination market characteristics and the within-firm skewness of export sales (after controlling for bilateral trade costs). Theoretically, this within firm change in product mix driven by the trading environment has important repercussions on firm productivity and how it responds to changes in that trading environment.

8 References

- Allanson, Paul and Catia Montagna (2005) Multiproduct Firms and Market Structure: An Exploratory Application to the Product Life Cycle, *International Journal of Industrial Organization* 23, 587-597.
- Arkolakis, Costas and Marc-Andreas Muendler (2010) The Extensive Margin of Exporting Goods: A Firm-level Analysis, University of California, San Diego, mimeo.
- Arkolakis Costas, Andres Rodriguez-Clare and Arnaud Costinot (forthcoming) New Trade Models, Same Old Gains? *American Economic Review*.
- Baldwin, John R. and Wulong Gu (2009) The Impact of Trade on Plant Scale, Production-Run

- Length and Diversification. In T. Dunne, J.B. Jensen and M.J. Roberts (eds.), *Producer Dynamics: New Evidence from Micro Data*, University of Chicago Press, Chicago.
- Bernard, Andrew B., Stephen J. Redding and Peter K. Schott (forthcoming) Multi-product Firms and Trade Liberalization, *Quarterly Journal of Economics*.
- Bernard, Andrew B., J. Bradford Jensen, Stephen J. Redding and Peter K. Schott (2007) Firms in International Trade, *Journal of Economic Perspectives* 21, 105-130.
- Eckel, Carsten and J. Peter Neary (2010) Multi-Product Firms and Flexible Manufacturing in the Global Economy, *Review of Economic Studies* 77(1), 188-217.
- Eaton, Jonathan, Samuel Kortum and Francis Kramarz (forthcoming) An Anatomy of International Trade: Evidence from French Firms, *Econometrica*
- Feenstra, Robert C. and Hong Ma (2008) Optimal Choice of Product Scope for Multiproduct Firms under Monopolistic Competition. In Helpman, Elhanan, Dalia Marin and Thierry Verdier (eds.) *The Organization of Firms in a Global Economy*, Harvard University Press, Cambridge MA.
- Feenstra, Robert C. (2004) *Advanced International Trade*, Princeton: Princeton University Press.
- Harrigan, James and Haiyan Deng (2010) China's Local Comparative Advantage. Chapter 3 in Robert C. Feenstra and Shang-Jin Wei (eds) *China's Growing Role in World Trade*, NBER Conference volume, Chicago University Press.
- Head, Keith and Thierry Mayer (2011) Gravity, Market Potential and Economic Development, *Journal of Economic Geography* 11(2): 281-294.
- Helpman E., M. J. Melitz and S. Yeaple (2004) Export Versus FDI with Heterogeneous Firms, *American Economic Review* 94, 300-316.
- Iacovone, Leonardo and Beata S. Javorcik (2008) Multi-product Exporters: Diversification and Micro-level Dynamics, *World Bank Working Paper No. 4723*.
- Mayer, Thierry and Gianmarco I.P. Ottaviano (2007) The Happy Few: The Internationalisation of European Firms, *Bruegel Blueprint* 3.
- Melitz, Marc J. and Gianmarco I.P. Ottaviano (2008) Market Size, Trade and Productivity, *Review of Economic Studies* 75, 295 -316.
- Nocke, Volker and Stephen R. Yeaple (2006) Globalization and Endogenous Firm Scope, *NBER Working Paper No. 12322*.
- Redding, Stephen .J. and Anthony J. Venables (2004) Economic Geography and International Inequality, *Journal of International Economics*, 62, 53-82.
- Wooldridge, Jeffrey W. (2006) Cluster sample methods in applied econometrics: An extended analysis. Michigan State University. Unpublished Manuscript. <http://www.msu.edu/~ec/faculty/wooldridge/current%20research.htm>

Appendix

A Tougher Competition and Firm Productivity

In Section 3 we argued that tougher competition induces improvements in firm productivity through its impact on a firm's product mix. Here we show that both firm productivity measures, output per worker $\Phi(c)$ and deflated sales per worker $\Phi_R(c)$, increase for all multi-product firms when competition increases (c_D decreases). We provide proofs for the closed as well as the open economy. In both cases we proceed in two steps. First, we show that, holding a firm's product scope constant, firm productivity over that product scope increases whenever competition increases. Then, we extend the argument by continuity to cover the case where tougher competition induces a change in product scope.

A.1 Closed Economy

Consider a firm with cost c producing $M(c)$ varieties. Output per worker is given by

$$\Phi(c) = \frac{Q(c)}{C(c)} = \frac{\sum_{m=0}^{M(c)-1} q(v(m, c))}{\sum_{m=0}^{M(c)-1} v(m, c) q(v(m, c))} = \frac{\frac{L}{2\gamma} \sum_{m=0}^{M(c)-1} (c_D - \omega^{-m} c)}{\frac{L}{2\gamma} \sum_{m=0}^{M(c)-1} \omega^{-m} (c_D - \omega^{-m} c)}.$$

For a fixed product scope M with $1 < M \leq M(c)$, this can be written as

$$\Phi(c) = \frac{\omega^M (\omega - 1) M c_D - \frac{c}{M} \frac{\omega(\omega^M - 1)}{\omega^M (\omega - 1)}}{\omega (\omega^M - 1) c \frac{c_D - c \frac{\omega(\omega^M + 1)}{\omega^M (\omega + 1)}}{c_D - c \frac{\omega(\omega^M + 1)}{\omega^M (\omega + 1)}}}, \quad (\text{A.1})$$

subject to $c \in [c_D \omega^M, c_D \omega^{M-1}]$. Differentiating (A.1) with respect to c_D implies that

$$\frac{d\Phi(c)}{dc_D} < 0 \iff c \frac{\omega (\omega^M + 1)}{\omega^M (\omega + 1)} > \frac{c}{M} \frac{\omega (\omega^M - 1)}{\omega^M (\omega - 1)}$$

or, equivalently, if and only if

$$M > \frac{(1 + \omega) (1 - \omega^M)}{(1 + \omega^M) (1 - \omega)}. \quad (\text{A.2})$$

This is always the case for $M > 1$: the left- and right-hand sides are identical for $M = 0$ and $M = 1$, and the right hand side is increasing and concave in M . This proves that, holding $M > 1$ constant, a firm's output per worker is larger in a market where competition is tougher (lower c_D).

Even when product scope M drops due to the decrease in c_D , output per worker must still

increase due to the continuity of $\Phi(c)$ with respect to c_D (both $Q(c)$ and $C(c)$ are continuous in c_D as the firm produces zero units of a variety right before it is dropped when competition gets tougher). To see this, consider a large downward change in the cutoff c_D . The result for given M tells us that output per worker for a firm with given c increases on all ranges of c_D where the number of varieties produced does not change. This just leaves a discrete number of c_D 's where the firm changes the number of products produced. Since $\Phi(c)$ is continuous at those c_D 's, and increasing everywhere else, it must be increasing everywhere.

The unavailability of data on physical output often leads to a measure of productivity in terms of deflated sales per worker. Over the fixed product scope M with $1 < M \leq M(c)$, this alternate productivity measure is defined as

$$\Phi_R(c) = \frac{R(c)/\bar{P}}{C(c)} = \frac{1}{2} \frac{k+2}{k+1} \frac{1}{c_D} \frac{M(c_D)^2 - c^2 \omega^2 \frac{\omega^{2M}-1}{\omega^{2M}(\omega-1)(\omega+1)}}{c_D c \omega \frac{\omega^M-1}{\omega^M(\omega-1)} - c^2 \omega^2 \frac{\omega^{2M}-1}{\omega^{2M}(\omega-1)(\omega+1)}}, \quad (\text{A.3})$$

subject to $c \in [c_D \omega^M, c_D \omega^{M-1}]$. Differentiating (A.3) with respect to c_D then yields

$$\frac{d\left(\frac{R(c)/\bar{P}}{C(c)}\right)}{dc_D} = -\frac{1}{2} \frac{k+2}{k+1} \frac{1+\omega^M}{1-\omega^M} \frac{M\omega^{2M}(1-\omega^2)(c_D)^2 - 2c\omega^{M+1}(1+\omega)(1-\omega^M)c_D + c^2\omega^2(1-\omega^{2M})}{(c_D)^2[\omega^M(\omega+1)c_D - c\omega(\omega^M+1)]^2} < 0.$$

Here, we have used the fact that $c \in [c_D \omega^M, c_D \omega^{M-1}]$ implies

$$M\omega^{2M}(1-\omega^2)(c/\omega^M)^2 - 2c\omega^{M+1}(1+\omega)(1-\omega^M)(c/\omega^M) > 0.$$

This proves that, holding $M > 1$ constant, this alternative productivity measure $\Phi_R(c)$ also increases when competition is tougher (lower c_D). The same reasoning applies to the case where tougher competition induces a reduction in product scope M .

Note that, in the special case of $M = 1$, we have

$$\Phi_R(c) = \frac{1}{2} \frac{k+2}{k+1} \left(\frac{1}{c} + \frac{1}{c_D} \right).$$

Hence, whereas tougher competition (lower c_D) has no impact on the output per worker $\Phi(c)$ of a single-product firm, it still raises deflated sales per worker $\Phi_R(c)$. This is due to the fact that deflated sales per worker are also affected by markup changes when the toughness of competition

changes.

A.2 Open Economy

Consider a firm with cost c selling $M_{lh}(c)$ varieties from country l to country h . Exported output per worker is given by

$$\Phi_{lh}(c) \equiv \frac{Q_{lh}(c)}{C_{lh}(c)} = \frac{\sum_{m=0}^{M_{lh}(c)-1} c_{hh} - \tau_{lh} \omega^{-m} c}{\sum_{m=0}^{M_{lh}(c)-1} (\tau_{lh} \omega^{-m} c) (c_{hh} - \tau^{lh} \omega^{-m} c)}.$$

For a fixed product scope M with $1 < M \leq M_{lh}(c)$, this can be written as

$$\Phi_{lh}(c) = \frac{\omega^M (1 - \omega)}{\omega (1 - \omega^M)} \frac{M}{c \tau_{lh}} \frac{c_{hh} - \frac{c \tau_{lh}}{M} \frac{\omega(1-\omega^M)}{\omega^M(1-\omega)}}{c_{hh} - c \tau_{lh} \frac{\omega(1+\omega^M)}{\omega^M(1+\omega)}}, \quad (\text{A.4})$$

subject to $c \tau_{lh} \in [c_{hh} \omega^M, c_{hh} \omega^{M-1}]$. Differentiating (A.4) with respect to c_{hh} yields

$$\frac{d\Phi_{lh}(c)}{dc_{hh}} < 0 \iff c \tau_{lh} \frac{\omega (\omega^M + 1)}{\omega^M (\omega + 1)} > \frac{c \tau_{lh} \omega (\omega^M - 1)}{M \omega^M (\omega - 1)}$$

This must hold for $M > 1$ (see (A.2)). Hence, tougher competition (lower c_{hh}) in the destination market increases exported output per worker. As in the closed economy, the fact that output per worker is continuous at a discrete number of c_{hh} 's and decreasing in c_{hh} everywhere else implies that it is decreasing in c_{hh} everywhere.

We now turn to productivity measured as deflated export sales per worker. Over the fixed product scope M with $1 < M \leq M(c)$, this is defined as

$$\Phi_{R,lh}(c) = \frac{R_{lh}(c)/\bar{P}_h}{C_{lh}(c)} = \frac{1}{2} \frac{k+2}{k+1} \frac{1}{c_{hh}} \frac{M (c_{hh})^2 - c^2 (\tau_{lh})^2 \omega^2 \frac{\omega^{2M}-1}{\omega^{2M}(\omega-1)(\omega+1)}}{c_{hh} c \tau_{lh} \omega \frac{\omega^M-1}{\omega^M(\omega-1)} - c^2 (\tau_{lh})^2 \omega^2 \frac{\omega^{2M}-1}{\omega^{2M}(\omega-1)(\omega+1)}}, \quad (\text{A.5})$$

subject to $c \tau_{lh} \in [c_{hh} \omega^M, c_{hh} \omega^{M-1}]$. Differentiating (A.5) with respect to c_{hh} yields

$$\frac{d\Phi_{R,lh}(c)}{dc_{hh}} = -\frac{1}{2} \frac{k+2}{k+1} \frac{1+\omega^M}{1-\omega^M} \frac{M \omega^{2M} (1-\omega^2) (c_{hh})^2 - 2c \tau^{lh} \omega^{M+1} (1+\omega) (1-\omega^M) c_{hh} + c^2 (\tau_{lh})^2 \omega^2 (1-\omega^{2M})}{(c_{hh})^2 [\omega^M (\omega+1) c_{hh} - c \tau_{lh} \omega (\omega^M+1)]^2} < 0.$$

The last inequality holds since $c\tau_{lh} \in [c_{hh}\omega^M, c_{hh}\omega^{M-1}]$ implies

$$M\omega^{2M} (1 - \omega^2) (c\tau_{lh}/\omega^M)^2 - 2c\tau_{lh}\omega^{M+1} (1 + \omega) (1 - \omega^M) (c\tau_{lh}/\omega^M) > 0.$$

This proves that, holding $M > 1$ constant, productivity measured as deflated export sales per worker increases with tougher competition in the export market (lower c_{hh}). The same applies to the case where the tougher competition induces a response in the exported product scope M , as $\Phi_{R,lh}(c)$ is continuous in c_{hh} .