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International Integration of Used Markets with a Monopoly Producer

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Abstract

We study the impact of restrictions on trade in used goods between countries in the presence of a monopolistic seller that sells in two different national markets. We provide a characterization of when movements towards trade liberalization will raise welfare and profits. Surprisingly, the monopolistic seller may prefer used markets to be liberalized and, instead, trade liberalization could be welfare reducing in the neighborhood of autarky if it exacerbates the monopoly distortion. In the case where trade is restricted by a quota, a necessary condition for profits to increase with trade in used goods is that the goods be imported to the country with the lower price of new goods. Trade is also more likely to be welfare improving when firm profits rise.

1 Introduction

Our goal in this paper is to address the impact of restrictions on trade in used goods between countries in the presence of imperfectly competitive markets for new goods. We examine the case of a monopoly seller of a new good selling in two different national markets, and facing a perfectly

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competitive market for used goods in each country. It is clear that the seller would like to be able to segment the market for new goods if possible, since this allows the possibility of raising profits by being able to charge different prices for new goods in the two markets. However, the question of whether the monopolist would also prefer to have restrictions on trade in used goods between markets has not been addressed.

This question is of interest because of the fact that trade in used goods has been a controversial issues in some recent trade agreements. For example, the market for used cars has been one of the markets where liberalization has been significantly delayed in the North American Free Trade Agreement. As recently as 2008, trade in used cars was restricted to cars that were more than 10 years old. The complete phase in of free trade in used cars is not scheduled to be completed until 2019. In the European Union, a report to the European Commission ¹ identified 9 countries for which imports of used cars accounted for 60% or more of new car registrations. In the case of Poland, this ratio has gone as high as 230% in some years. A number of these countries have attempted to impose inspection and technical restrictions on imported used cars that do not apply to domestic vehicles, resulting in litigation at the European Court of Justice. Our model is capable of addressing whether restrictions on trade in used goods can be welfare improving when the new goods market is monopolized.²

Our modeling draws on the existing theoretical literature that has developed to understand the role of markets in used durables. This literature has generally used buyer heterogeneity to explain the existence of markets for used goods.³ Used goods contain a different bundle of attributes than new

 $^{^1 \}mathrm{See}$ Melhart, Merz, Akkermans, and Jordal-Jorgenson (2011)

²Another issue that has been raised by trade in used goods is its environmental impact. Davis and Kahn (2010) empirically study NAFTA's environmental implications and obtain a negative finding. Surprisingly, their finding does not rely on the characteristics of the cars being imported into Mexico-indeed, these cars are less polluting than the existing car fleet, but on the lengthening of the imported cars' life expectancy. We abstract from the environmental impact in our analysis.

³Our work also indirectly relates to the literature seeking to understand the effects of opening secondary markets (see Anderson and Ginsburgh (1994), Chen, Esteban, and Shum (2012), Esteban and Shum (2007), Johnson (2011), and Porter and Sattler (1999) among others). Opening secondary markets and unifying them are two substantially different problems. When we allow for trade between countries, secondary markets already exist. The difference is that trade places together consumers who may derive different usefulness from the consumption of used goods and have different valuations. In nature, our problem is one of third degree price discrimination—solving the trade-off between segmenting or unifying demands—and not one of second degree which is what the firm indirectly achieves when it opens a secondary market.

goods: they generally provide a lower quality of service than new goods, and require greater amounts of maintenance as they are more prone to breakdowns. A used market thus allows goods to be passed from consumers who value the attributes of new goods most highly to those who place a higher relative valuation on the attributes of used goods as the goods age. Some of the earliest literature on this issue arose from the observation of a substantial trade in used goods between countries, where differences in factor prices across countries provided a natural source of buyer heterogeneity. Sen (1962) explained how differences in wage across countries, which help determine maintenance and repair costs, can result in gains from the trade in used goods. Smith (1974) analyzed the efficient international allocation of capital goods in the presence of factor price differences, and showed that the high wage country should specialize in the use of new machines. Grubel (1980) explains how the slower depreciation rate of goods in developing countries can cause gain from trade.⁴

There is also substantial within country heterogeneity of buyers that can be used to explain the existence of used trade within countries. Differences in the willingness to pay for quality across consumers will result in gains from the formation of used markets, with used goods allocated to those with a low willingness to pay for quality. Income differences across consumers are a natural source of such heterogeneity. Gavazza, Lizzeri, and Roketskiy (2012) empirically assess (with what would be an autarky model) the implications of differences in the income distribution on secondary market behavior. They evaluate the cases of France and the United States-the former being a country with a much less disperse income distribution-and show that, although new car prices would be higher in the U.S., the price decline as the car ages would be less pronounced in France. Bond (1983) analyzes data from purchases of new and used trucks, and finds evidence to support a model in which new goods are more valuable to buyers with greater intensity of use and higher costs of maintenance.

Our modeling of the used market assumes two sources of buyer heterogeneity. One is due to differences in consumer incomes within a country, which results in a separation in which high income buyers choose new goods and low income buyers choose used goods. A second source of heterogeneity arises from the potential for differences in the rate at which the quality of services of the durable declines across countries. This feature is motivated by the observations from the international trade literature that lower maintenance

⁴Trade in used goods can also be a result of quality regulations, as studied by Clerides and Hadjiyiannis (2008)

and higher capital costs may make used goods relatively more attractive in high income countries. We show that this modeling approach can give rise to two factors that generate differences in prices of used goods across countries, and hence potential gains from trade in used goods between countries. One arises from differences in the mean level of income across countries and its impact on monopoly markups, and the other from differences in the value of used goods due to differences in the relative decline in the value of services provided.

Section 2 of the paper analyzes the case in which the monopolist can commit to future output levels in autarky for varying degrees of new market and used market integration. We begin with the case in which both new and used markets are segmented. We show that if the rate of depreciation in quality of services is similar between countries, then the richer country will have higher prices of both new and used goods at autarky. However, the poorer country could have a higher price for used goods if its rate of quality depreciation is sufficiently high relative to the richer country. We then examine the effect of a small trade liberalization in the neighborhood of autarky, and show that a necessary condition for the monopolist to gain from trade in used goods is that the goods flow from the high income to the low income country. Since the flow of goods will switch sales of new goods from the importer of used goods to the exporter, this trade is more likely to be profitable to the monopolist if the importer has a low price of new goods. We also show that a sufficient condition for trade in used goods to raise welfare is that it reduces the monopoly distortion. A sufficient condition for this case is that the monopolist earns higher profits as a result of the trade in used goods.

We also examine the case in which used markets are segmented, but used markets are completely integrated. An interesting feature of this case is that the direction of trade is completely determined by the rate of quality depreciation in the two countries. Thus, autarky price differences may be poor indicators of the direction of trade in used goods. Finally, we consider the case of perfectly integrated markets for new goods, but quantity restricted trade in used goods. We show that in this case, the monopolist will only gain from trade in used goods if the goods are flowing to the location where the depreciation rate of used goods is higher.

We conclude our analysis by considering the case in which the monopolist is unable to commit to future output levels, so that the optimal production plan must be time-consistent. We show that for the case in which new markets are segmented and there is a quantitative restriction on used trade, the time consistent solution yields higher outputs in each country than does the solution with full commitment. We also use a simulation to show that the conclusions regarding the pattern of trade and the profitability of trade liberalization in the used good market under time consistency are similar to those in the case with commitment.

2 Model

We examine a two country model in which there is a monopoly seller of a durable good who sells the durable good in both countries. The durable good is assumed to last for two periods, and to generate a service flow of 1 unit in the first period of its life and q < 1 in the second period in the home country. There is a fixed number of infinitely-lived consumers (normalized to 1). Consumers are heterogeneous and their type is identified by their willingness to pay for a unit of quality, θ : a consumer of type θ receives a value of θ from use of a new good for one period and θq from the use of a used good. The taste parameter is assumed to be uniformly distributed on the interval [0, b], so G(x) = b(1 - x) denotes the taste parameter of the (1 - x) 100 percentile of the taste distribution.

Tastes in the foreign country are uniformly distributed on the interval $[0, b^*]$, where $b^* < b$. Letting $G^*(x) = b^*(1-x)$, this ensures that the taste for services of new goods of the (1 - x) 100 percentile of consumers will always be higher in the home country. This can be interpreted as reflecting a higher level of income in the home country than in the foreign country. We will also allow for the possibility that the depreciation rate of services is specific to the country of consumption, so that q^* does not necessarily equal q. In particular, there is considerable empirical evidence that the depreciation rate of services. This could be due to a lower price of repair services in developing countries, or to a relatively higher cost of capital.

In this section we focus on the case where the monopolist can commit at time 0 to a path of output levels for new goods in each market, $\{x_t, x_t^*\}_{t=1}^{\infty}$. Used markets are assumed to be perfectly competitive in each country.

2.1 Equilibrium with Segmented New Markets and a Quota on Trade in Used Goods

We begin by characterizing the equilibrium when the seller is able to effectively segment the market for new goods, and trade in used goods is assumed to be subject to a quantitative restriction⁵. Since the direction of trade will be determined by the home and foreign taste parameters, we have two possibilities. If the home country is an importer of used goods, then m > 0 will denote the home country import quota that restricts home imports to be no greater than m. If the home country is an exporter of used goods, then -m > 0 The case of m = 0 represents autarky, in which there is no trade in new or used goods. We will use the solution at the autarky equilibrium to determine the pattern of trade between the countries, and then examine how allowing quota-restricted trade in used goods in the neighborhood of the autarky equilibrium affects the profits of the monopolist and welfare. Since our focus in this section is on behavior in the neighborhood of the autarky equilibrium, we assume that the quantitative restriction is binding.

We will assume that buyers correctly anticipate the future resale value of used goods at t + 1, which will ensure that the new goods will be allocated to consumers of type $\theta \in [G(x_t), b]$ and the used goods to consumers of type $\theta \in [G(x_{t-1} + x_t + m), G(x_t))$. In the foreign country, consumers with $\theta \in [G^*(x_t^*), b^*]$ ($\theta \in [G^*(x_{t-1}^* + x_t^* - m), G^*(x_t^*))$) buy new (used) goods. Since the price of used goods must be such that the marginal buyer of used goods is indifferent between a new good and a used good, the inverse demand function for used goods in the respective countries will be

$$p_U(x_{t-1} + x_t + m) = qG(x_{t-1} + x_t + m).$$
(1)

$$p_U^*(x_{t-1}^* + x_t^* - m) = qG(x_{t-1}^* + x_t^* - m).$$
(2)

The assumption of a binding quota will hold as long as $p_U(x_{t-1} + x_t + m) \ge p_U^*(x_{t-1}^* + x_t^* - m)$ when m > 0 or $p_U(x_{t-1} + x_t + m) \le p_U^*(x_{t-1}^* + x_t^* - m)$ when $m^* < 0$. We will assume that the rights to trade used goods under a quota are allocated to third parties in the importing country, such as dealers in used durable goods. The holders of these licenses will capture the quota rents, $(p_U(x_{t-1} + x_t + m) - p_U^*(x_{t-1}^* + x_t^* - m))m$.

The price of new goods will be such that the marginal buyer is indifferent between buying new and buying used, where the value of a new good includes the resale in the used market in the following period. Letting $E_t x_{t+1}$ denote the expectation of consumers at time t of sales at t + 1 and β the discount factor, a consumers will prefer a new good at time t if $\theta + \beta E_t p_{Ut+1} - p_{Nt} \ge$

⁵Even in the absence of barriers to trade in new goods, a monopolist may be able to segment markets for new goods through the use of country-specific warranties that make it difficult to engage in arbitrage in new goods. Also, arbitrage will not be possible if the good is patented and the countries follow a policy of national exhaustion of patent rights, prohibiting parallel trade.

 $\max\{\theta q - p_{Ut}, 0\}$ and will prefer a used good if $\theta q - p_{Ut} \ge \max\{\theta - p_{Nt} + \beta E_t p_{Ut+1}, 0\}$. The inverse demand functions for new goods will be

$$p_N = G(x_t) (1 - q) + \beta p_U(x_t + E_t x_{t+1} + m) + p_U(x_{t-1} + x_t + m)$$

$$p_N^* = G^*(x_t^*) (1 - q^*) + \beta p_U(x_t^* + E_t x_{t+1}^* - m) + p_U(x_{t-1} + x_t - m)$$
(3)

The inverse demand functions ensure that consumer type $G(x_t)$ will be indifferent between new and used and consumer with type $G(x_t + x_{t-1})$ is indifferent between used and being out of the market. Note that the in the presence of a binding quota, changes in the quantity of goods in one market will have no effect on the prices of new and used goods in the other market.

In light of the independence of prices across markets, the profit maximization problem for the home country market can be analyzed independently of the foreign country market. With commitment to output levels by the monopolist, the discounted profits of the monopolist can be expressed as

$$\Pi(x_0) = \max_{\{x_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} (p_N(x_{t-1}, x_t, x_{t+1}) - c) x_t$$
(4)

where the assumption of commitment by the monopolist ensures that $E_t x_{t+1} = x_{t+1}$. The necessary condition for the choice of x_t is

$$(p_N(x_{t-1}, x_t, x_{t+1}) - c + x_t G'(x_t)(1-q) + (x_t + x_{t+1}) \beta q G'(x_t + x_{t+1} + m) + (x_t + x_{t-1}) q G'(x_t + x_{t-1}) = 0$$

Substituting into this relationship from the definition of G for the case of a uniform distribution yields

$$(1 + \beta q - c/b - mq(1 + \beta)) - 2(qx_{t-1} + (1 + \beta q)x_t + \beta qx_{t+1}) = 0$$
 (5)

This difference equation has a solution of the form $x_t = A_0 + A_1 x_{t-1}$. Solving (5) yields $A_0 = \frac{(1+\beta q - mq(1+\beta) - c/b)(1+3q\beta - \sqrt{1+2q\beta - q^2(4-\beta)\beta})}{4q\beta(1+q+2\beta q)}$ and $A_1 = \frac{-1-q\beta + \sqrt{1+2q\beta - q^2(4-\beta)\beta}}{2q\beta}$. The steady state output will be A_0 which yields

The steady state output will be $\frac{A_0}{1-A_1}$, which yields

$$x^{SS} = \frac{1 + \beta q - mq(1+\beta) - c/b}{2(1+q+2\beta q)}$$
(6)

In order for the steady state to have a positive output of durables, the value of the service flow from the durable to the highest valuation buyer must be higher than its cost, $b(1 + \beta q) > c$. The steady sate output will be increasing in *b*, since larger *b* is associated with a higher average valuation on services from the durable. An increase in the quantity of imports will reduce the steady state output, since the import of used goods will be a substitute for new goods. Finally, an increase in *q* will reduce the steady state output level because it reduces the elasticity of demand for new goods.

Substituting the steady state output level into (1) - (4) yields the steady state output, price and profits for the home country:

$$p_{N}^{SS} = \frac{b + \beta bq + c - mq(1 + \beta)}{2}$$

$$p_{U}^{SS} = \frac{q \left(bq(1 + \beta) + c - bm(1 + q\beta)\right)}{1 + q + 2\beta q}$$

$$\pi^{SS} = \frac{\left(b(1 + \beta q - mq(1 + \beta)) - c\right)^{2}}{4b \left(1 + q + 2\beta q\right)}$$
(7)

The prices of both new and used goods are increasing in b and q and decreasing in m. Steady state values for the foreign country are derived in a similar fashion and are given by:

$$p_N^{*SS} = \frac{b^* + \beta b^* q^* + c + mq^*(1+\beta)}{2}$$

$$p_U^{*SS} = \frac{q^* (b^* q^*(1+\beta) + c + b^* m(1+q^*\beta))}{1+q^* + 2\beta q^*}$$

$$\pi^{*SS} = \frac{(b^*(1+\beta q^* + mq^*(1+\beta)) - c)^2}{4b^* (1+q^* + 2\beta q^*)}$$

$$x^{*SS} = \frac{1+\beta q^* + mq^*(1+\beta) - c/b^*}{2(1+q^* + 2\beta q^*)}$$
(8)

The signs of the effects of imports are reversed for the foreign steady state values, because an increase in m is a reduction in the supply of used goods in the foreign country.

2.1.1 The Pattern of Trade and Profits

We can use (7) to compare the prices of new and used goods in the autarkic steady state. Price of new goods will be higher in the home country at autarky if $b(1 + q\beta) > b^*(1 + q^*\beta)$. This NN locus in Figure 1 illustrates the values of b^* and q^* at which goods prices are equalized to those in the home country, given b and q. Prices of new goods will be higher in the home country for all values of (b^*, q^*) below the NN locus (regions **A**, **B**,



Figure 1: Comparative Advantage and Profitability of Trade Liberalization with Segmented New Markets and Quo $(b = 1.8, q = .5, \beta = .9, c = 1)$

D in Figure 1). Prices of used goods will be higher in the home country for $\frac{q(bq(1+\beta)+c)}{1+q+2\beta q} > \frac{q^*(b^*q^*(1+\beta)+c)}{1+q^*+2\beta q^*}$. The UU locus in Figure 1 illustrates the values of b^* and q^* at which used prices are equalized across countries: the home country will have a higher price of used goods for values of (b^*, q^*) below the UU locus (regions **A**, **C**, **E**). If $q = q^*$, the home country will have a higher price of both new and used goods if $b > b^*$. Note that the NN locus must be flatter than the UU locus, because q has a relatively larger impact on prices of used goods than on prices of new goods. For values of the parameters above or below both of the loci, the country with the higher price of used goods also has a higher price of new goods. For parameter values between the two loci, the country with the higher price of used goods. The region between the two loci has characteristics that are associated with observed trade in used goods, because high income countries (i.e. those with a higher upper bound of the taste parameter) are exporters of used goods.

For parameter values below the UU locus, the home country has a comparative advantage in used goods. Trade liberalization in the neighborhood of autarky will involve dm > 0. For parameters above the UU locus, the foreign country has comparative advantage in used goods and trade liberalization will require dm < 0. We can determine the effect of trade liberalization on the global profits and evaluating at m = 0, which yields

$$\frac{d\pi^{SS}}{dm} + \frac{d\pi^{SS*}}{dm}\Big|_{m=0} = (1+\beta) \left(q^* b^* x^{SS*} - q b x^{SS}\right)' \tag{9}$$

The Π^W locus shows the values of b^* and q^* at which trade liberalization has no effect on profits and must lie between the NN and UU loci. For values below (above) the Π^W locus world profits are decreasing (increasing) in m.

We can use these results to identify the impact of trade liberalization on monopoly profits in each of the 6 regions in Figure 1. We begin with regions A, B, and D where the home country has a higher autarkic price of new goods. The results for the regions where the foreign country has a higher price of new goods are symmetric. In region A, the home country has a higher price of both new and used goods, and the home country will be an importer of used goods. Profits of the monopolist are decreasing in m in this region, so the monopolist must lose from trade liberalization because it will involve dm > 0. Trade harms monopoly profits because the flow of used goods into the home country reduces sales in the firm's high price market and raises sales in its low price market. In region **B**, the home country will have a higher price of new goods but a lower price of used goods. Trade liberalization will result in a flow of used goods to the foreign market, dm < 0, which will raise the profits of the monopolist in this region because world profits are decreasing in m. In this case the flow of used goods is reducing sales in the monopolist's low price market and raising them in the high price market, which raises profits. The pattern of trade in region **D** is similar to that in **B**, so trade liberalization will imply dm > 0. The difference between regions \mathbf{B} and \mathbf{D} is that profits are increasing in m in region **D**, so the monopolist's profits will be reduced by used trade in this region as well. In regions C, E, and F the foreign country has the higher price of new goods, so the results are similar but with the identity of the countries reversed. The monopolist earns higher profits with trade liberalization in used goods in regions \mathbf{B} and \mathbf{E} , but earns lower profits in the other regions. This establishes that a pattern of trade in which used goods flow to the market with the higher price of new goods (which occurs in B, C, D, E) is necessary but not sufficient for trade in used goods to raise the monopolist's profits.

2.1.2 Trade and Welfare

We now turn to the effect of trade liberalization on welfare in the neighborhood of the steady state. A consumer purchasing a new good at time t in the home country and reselling in the next period receives a surplus of $\theta - p_{Nt} + \beta p_{Ut}$ from the use of the good while new. A consumer purchasing a used good at home receives a surplus of $\theta q - p_{Ut}$. Aggregating over buyers of new and used goods and summing over all period yields a consumer surplus in the home country of

$$S = \sum_{t=0}^{\infty} \left[\int_{b(1-x_t)}^{b} \frac{\theta}{b} d\theta + \int_{b(1-x_t-x_{t-1}-m)}^{b(1-x_t)} \frac{q\theta}{b} d\theta - p_{Nt} x_t - p_{Ut} m \right] \beta^t$$

Totally differentiating this expression and using the conditions for optimal consumer choice of used goods yields

$$dS = -\sum_{t=0}^{\infty} \left(x_t dp_{Nt} + m dp_{Ut} \right) \beta^t$$

Consumer welfare will be decreasing in the price of new goods, so trade liberalization in the used good market will benefit consumers if it results in a reduction in the price of new goods. From (7), home consumers will receive a net benefit from trade liberalization in the steady state iff the home country is an importer of used goods. Increases in the price of used goods result in a redistribution in surplus between buyers of new goods and buyers of used goods. The net effect of an increase in the price of used goods on consumer welfare will be negative (positive) if the home country is an importer (exporter) of used goods. Trade liberalization will have a favorable impact on consumers through the price of used goods if it is an importer

For the monopolist, we can totally differentiate (4) to obtain the change in profits from the home market to be $d\Pi = \sum_{t=0}^{\infty} ((p_{Nt} - c) dx_t + x_t dp_{Nt}) \beta^t$. Adding together the change in consumer and producer surplus in each country yields

$$dS + d\Pi = \sum_{t=0}^{\infty} \left((p_{Nt} - c) \, dx_t + m dp_{Ut} \right) \beta^t.$$
(10)
$$dS^* + d\Pi^* = \sum_{t=0}^{\infty} \left((p_{Nt}^* - c) \, dx_t^* - m dp_{Ut}^* \right) \beta^t$$

The change in consumer and producer welfare consists of two components. The first reflects the inefficiency associated with monopoly: an increase in output of new goods in a market will increase surplus because the monopolist's price exceeds marginal cost. The second is the terms of trade effect on used goods, which shows that an increase in the price of used goods will benefit an exporting country and harm an importing country. Note that changes in prices of new goods merely redistribute income between consumers and producers, and thus do not appear in the welfare expression.

The one remaining component in the welfare calculation is the rent collected from the quota, $R = \sum_{t=0}^{\infty} (p_{Ut} - p_{Ut}^*) m\beta^t$. Differentiating yields $dR = \sum_{t=0}^{\infty} ((p_{Ut} - p_{Ut}^*) dm + m (dp_{Ut} - dp_{Ut}^*)) \beta^t$. The first term is the effect of changes in trade volume at a given license value, $(p_{Ut} - p_{Ut}^*)$, while the second is the impact of changes in the value of a quota license. The impact of changes in quota rents on national welfare will depend on how they are allocated between countries. Therefore, we begin by examining the change in world welfare, which sums together the producer and consumer gains in (10) with the change in quota rents:

$$dW^W = \sum_{t=0}^{\infty} \left((p_{Nt} - c) \, dx_t + (p_{Nt}^* - c) \, dx_t^* + (p_{Ut} - p_{Ut}^*) \, dm \right) \beta^t \tag{11}$$

Changes in prices of new goods in (10) result in redistribution between buyers of used goods and holders of quota licenses, and thus do not affect aggregate world welfare. The last term in (11) is the standard gains from trade liberalization: increases in the volume of trade in used goods will raise welfare because it moves goods from a low value location to a high value allocation. The first two terms represent the impact of changes in outputs of new goods on welfare due to the monopoly distortion in each country. Note that there is an important second best component to trade liberalization in this setting because of the monopoly distortion. Increases in the volume of imports will raise output of new goods in the exporting country and reduce the output of new goods in the importing country. These changes in new good sales could be either welfare increasing or welfare reducing depending on the magnitudes of the changes in sales and the magnitudes of the markups in each market.

Using (11), the effect of trade liberalization on welfare at state will be $\frac{dW^W}{dm}(1-\beta) = (p_N^{SS}-c)\frac{dx^{SS}}{dm} + (p_N^{SS}-c)\frac{dx^{*SS}}{dm} + (p_U^{SS}-p_U^{*SS})$. We can evaluate the welfare effect of trade liberalization in the neighborhood of autarky by substituting from (7) and evaluating at m = 0. Using the fact that $(p_N^{SS}-c)\frac{dx^{SS}}{dm} = \frac{1}{2}\frac{d\pi^{SS}}{dm}$ at m = 0, the expression for welfare change

can be written as

$$\frac{dW^W}{dm}(1-\beta) = \frac{1}{2} \left(\frac{d\pi^{SS}}{dm} + \frac{d\pi^{SS*}}{dm} \Big|_{m=0} \right) + \left(p_U^{SS} - p_U^{*SS} \right)$$

Since the last term will always be positive, a sufficient condition for trade liberalization to improve welfare is that the monopolist's profits increase. Therefore, trade liberalization must increase welfare in regions \mathbf{B} and \mathbf{E} in Figure 1.

For the remaining regions in Figure 1, the impact on world welfare will depend on the relationship between the size of the gains from trade in used goods against the losses in profits. As an example, consider the case where $q = q^*$, so that consumers differ only in the taste parameters. Evaluating the welfare change for this case, $\frac{dW^W}{dm}(1-\beta) = \frac{(b^*-b)q(1+2\beta)(1+q(4-\beta))}{4(1+q+2\beta q)}$. Trade liberalization will raise welfare in this case iff $q > \frac{1}{4-\beta}$. When q is sufficiently low, the gains from trade in used goods will be sufficiently small that they are dominated by the decline in profits.

2.2 Segmented New Markets with Free Trade in Used Goods

We now extend the analysis of trade in used goods to the case in which the quota is sufficiently large that it does not bind. With free trade in used goods, the price of used goods will be equalized across countries. Letting p^{UW} denote the price of used goods on the world market, the consumer who is indifferent between used goods and being out of the market in the home (foreign) country is the one with willingness to pay p^{UW}/q (p^{UW}/q^*). Since new goods are not traded, consumers of new goods will be those with $\theta \in [G(x_t), b]$ at home and $\theta \in [G^*(x_t^*), b^*]$ in the foreign country. Consumers of used goods will be those with $\theta \in [p^{UW}/q^*, G^*(x_t^*)]$ in the foreign country. The market clearing price for used goods will be the one at which the demand for used goods equals the supply, $x_{t-1} + x_{t-1}^*$. At an interior solution with consumption of used goods in both countries, we have

$$p_U^W = \frac{bb^* q q^* (2 - Y_t^W)}{bq + b^* q^*},\tag{12}$$

where $Y_t^W = x_{t-1} + x_{t-1}^* + x_t + x_t^*$ is the worldwide supply of the durable goods, both new and used.

The inverse demand function for new goods is obtained by substituting

(12) into (3) evaluated at m = 0, which yields

$$p_N(x_t, Y_t, E_t Y_{t+1}) = G(x_t) (1-q) + \beta p^U(E_t Y_{t+1}) + p^U(Y_t)$$
(13)
$$p_N^*(x_t^*, Y_t, E_t Y_{t+1}) = G^*(x_t^*) (1-q) + \beta p^U(E_t Y_{t+1}) + p^U(Y_t)$$

Increases in output in one market will reduce the price of used goods, which reduces the willingness to pay for new goods in each market. This spillover effect creates a linkage between sales in the two markets, even though the new goods markets are segmented. The global profits of the monopolist for this case will be

$$\Pi(Y_0) = \max_{\{x_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} \left[(p_N(x_{t-1}, Y_t, Y_{t+1}) - c) x_t + (p_N(x_{t-1}^*, Y_t, Y_{t+1}) - c) x_t^* \right]$$
(14)

In a steady state with $x_t = x$ and $x_t^* = x^*$,⁶ the profit maximizing solution to (14) is equivalent to choosing x and x^* to maximize

$$\pi = [G(x)x + G^*(x^*)x^*] (1 - q) + [(1 + \beta q) p_U(2(x + x^*)) - c] (x + x^*)$$
(15)

The first term in the profit function shows that the willingness to pay of the marginal buyer of new goods will not be equalized across markets, due to the segmentation of the new good markets. However, quality adjusted willingness to pay for used goods will be equated across markets due to the presence of trade in used goods.

The profit maximizing steady state output levels are with integrated used markets will be

$$\begin{aligned} x^{SI} &= \frac{b^2(1-q)q(1+q^*+2q^*\beta)+(1-q^*)(bb^*q^*(1-q)-(bq+b^*q)^*c)}{2b(b^*(1-q^*)q^*(1+q+2q\beta)+b(1-q)q(1+q^*+2q^*\beta))} \\ x^{*SI} &= \frac{b^{*2}(1-q^*)q^*(1+q+2q\beta)+(1-q)(bb^*q(1-q^*)-(bq+b^*q)^*c)}{2b^*(b^*(1-q^*)q^*(1+q+2q\beta)+b(1-q)q(1+q^*+2q^*\beta)} \end{aligned}$$

Since the expressions for output are substantially more complex than in the case with quotas, it is useful to begin by deriving results for the case in which countries differ only in the distribution of tastes: $b > b^*$, $q = q^*$.

2.2.1 Countries Differ Only in Taste Distribution

The discussion of comparative advantage in the previous section established that the foreign country would have a lower price of used goods when $b > b^*$,

⁶We can show that maximizing the one-period profit with respect to the steady state output is equivalent to maximizing the discounted sum of profits with respect to future output and evaluating the necessary condition at the steady state.

 $q = q^*$. The following result summarizes the impact of trade on the steady state output, prices, and profits:

Proposition 1 (Profit Maximizing Output at an interior solution with $b > b^*, q = q^*$)

(i) The effect of opening trade in used goods is to raise output in the home country and reduce output in the foreign country, with world output unaffected. The difference between output with trade in used goods and output under autarky is

$$x^{SS} - x^{SI} = \frac{(b - b^*)q(1 + \beta)}{2(b + b^*)(1 + q(1 + 2\beta))} = -\left(x^{*SI} - x^{*SS}\right) > 0$$

Prices of goods new and used goods will fall in the home country as a result of trade, and prices in the foreign country will rise. There will be no trade in used goods in the steady state.

(ii) The profits of the monopolist fall with trade in used goods relative to the autarky profit levels. The difference in worldwide profits is

$$\pi^{SI} - \pi^{SS} = -\frac{((b-b^*)q(1+\beta))^2}{4(b+b^*)(1+q(1+2\beta))} < 0$$

(iii) World welfare in the steady state rises with trade in used goods. The difference in welfare between the trade and autarkic steady states is

$$W^{ST} - W^{SA} = \frac{(b - b^*)^2 q(1 + \beta)(2 + q^2(1 - \beta) + 1(1 + 5\beta))}{8(b + b^*)(1 + q(1 + 2\beta))^2}$$

In markets for non-durable goods, the elimination of market segmentation will have no effect on world output when demand curves are linear and both markets are being served without market segmentation. Part (i) of the Proposition suggests a similar result, in that world output is unaffected by the introduction of trade in used goods between markets. Note however that in the case of the durable good, the opening of trade in used goods does not eliminate price differences in the market for new goods. The country with a higher average willingness to pay will have a higher price. Interestingly, however, this price difference in new goods does not result in trade in used goods. Although the autarkic price difference would suggest that the home country should be an importer of used goods, the output adjustment in new goods following the opening of trade eliminates any gains from trade.

Proposition 1 also shows that the monopolist will oppose trade in used goods in the case where $q = q^*$, since it will reduce profits by reducing the



seller's ability to segment markets. This result is the same as obtained for the case of liberalization in the neighborhood of free trade using a quota. The reallocation of output from the low valuation market to the high valuation market that results from trade in used goods will raise world welfare. If the taste difference between the home and foreign countries is sufficiently large, it could result in the elimination of sales of new goods in the low valuation market.

2.3 The General Case with Free Trade in Used Goods

We now turn to establishing some results for the general case. One simple result that emerges concerns the pattern of trade in used goods. The results with $q = q^*$ showed that the monopolist's pricing of new goods would result in the elimination of trade in used goods when there is free trade in used goods. For the general case, home imports of new goods in the steady state under free trade will be

$$m = \frac{(q-q^*)\left((bq+b^*q^*)c+b^*q^*bq(1+\beta)\right)}{(bq+b^*q^*)\left(b^*(1-q^*)q^*(1+q+2q\beta)+b(1-q)q(1+q^*+2q^*\beta)\right)}$$

The foreign country will import used goods iff $q^* > q$. The pattern of trade in used goods is determined solely by which country has the higher valuation on the services of used goods. Thus, the pattern of trade can be reversed from that predicted by prices of used goods in the autarkic steady state.

A second question concerns the profits obtained by the monopolist when there is free trade in used goods as opposed to the case under autarky. In Figure 1, the regions **B** and **E** were associated with an increase in profits due to trade liberalization. This region was contained in the region where the country with a higher price of new goods was an exporter of used goods. Figure 2 identifies the relationship between (b^*, q^*) and the change in profits under free trade using the same parameter values as in Figure 1. The profits of the monopolist are higher under free trade than in autarky in regions **B** and C, and are lower in the remaining areas A and D. The comparison between the regions where trade liberalization in used markets benefits the monopolist between Figures 1 and 2 illustrates two interesting points. One is that the area under which the monopolist benefits from trade is larger under free trade than under a small amount of liberalization with a quota. The second is that the comparison of autarkic prices for used goods provides a much better indicator of whether the monopolist will gain from free trade in used goods than does the pattern of trade at free trade.

2.4 Integrated New Markets with Quotas on Used Goods

We conclude our analysis of the case with commitment by examining the case where there is free trade in new goods but trade in used goods is restricted by a quota. Since the used goods are restricted by a quota, the price of used goods will be determined by (1) as in the case with segmented used markets. The difference with an integrated market for new goods is that the allocation of new goods to the respective markets, x_t and x_t^* , will not be chosen by the monopolist as it was in the case of segmented markets. The monopolist will produce an output of $x^W = x_t + x_t^*$ for the world market, with the allocation between countries determined to equalize the prices of new goods.

In a steady state where the seller produces a quantity x^W in each period, the price of new goods must satisfy

$$p_N = G(x) (1-q) + (1+\beta)qG(2x+m).$$
(16)

$$p_N = G^*(x^W - x) \left(1 - q^*\right) + (1 + \beta)q^*G(2(x^W - x) - m)$$
(17)

This pair of equations can be solved for p_N and x, obtaining

$$p_N^W(x^W,m) = \frac{\Gamma_0 - \Gamma_1 x^W + bb^*(1+\beta)(q-q^*)m}{b(1+q+2q\beta) + b^*(1+q^*+2q^*\beta)}$$
$$x(x^W,m) = \frac{b(1+q\beta) - b^*(1+q^*\beta) + b^*(1+q^*+2q^*\beta)x^W + bb^*(1+\beta)m}{b(1+q+2q\beta) + b^*(1+q^*+2q^*\beta)}$$

where

$$\Gamma_0 = bb^* (2 + (q + q^*) (1 + 3\beta) + 2qq^* (1 + 2\beta) > 0 \Gamma_1 = bb^* [(1 + q^* + 2q^*\beta)(1 + q + 2q\beta)] > 0$$

The world price is decreasing in the monopolist's output, and increasing in home imports iff $q > q^*$. The home country market will receive a share $\frac{b^*(1+q^*+2q^*\beta)}{b(1+q+2q\beta)+b^*(1+q^*+2q^*\beta)} \text{ of total output.}$ The profits of the monopolist in the steady state will be

$$\Pi^W = (p_N^W(x^W, m) - c)x^W$$

This yields a steady state output with integrated new markets and segmented used markets of

$$x^{WIS} = \frac{\Gamma_0 - bb^*(1+\beta)(q-q^*)m}{2\Gamma_1}$$
(18)

It is straightforward to show using (18) and the solutions (6) and (7) that world output in the autarkic steady state is the same as world output when there is trade in new goods but no trade in used goods.

The solution for steady state output in (18) can be used to solve for the prices and profits in the steady state. The UU locus in Figure 3 shows the locus of values of (b^*, q^*) at which prices of used goods are equalized, using the same parameter values as in Figure 1. The point to note here is that the UU locus is positively sloped when markets for new goods are integrated, but negatively sloped in the case of segmented new markets in Figure 1.

Profits from the world market in the steady state will be equal to $\Gamma_1 (x^{WIS})^2$ The effect of trade liberalization in the neighborhood of autarky will be $\frac{d\Pi^W}{dm} = -x^{WIS} \left(\frac{bb^*(1+\beta)(q-q^*)}{2\Gamma_1} \right), \text{ which will be positive iff } q^* > q. \text{ Trade}$ liberalization in used goods will only be good for the monopolist in the neighborhood of autarky if it results in the movement of goods to the location where the value placed on services is lower. Referring to Figure 3, trade liberalization will raise profits of the monopolist in regions **B** and **C**.

3 Time Consistency

In this section we analyze how the results are altered if the monopolist is unable to commit to future output levels. Since increases in the supply of new goods depress the price of used goods, a producer maximizing at t+1 will have an incentive to produce an output level that exceeds the



output level x_{t+1} that was derived in the full commitment solution. Forward looking consumers will anticipate the future actions of producers, and adjust $E(x_{t+1})$ accordingly. Letting g(y) denote the monopoly seller's decision rule for production as a function of the current stock of used goods, y, a time consistent solution will be one where this decision rule maximizes firm profits (4) given that $E(x_{t+1}) = g(x_t)$. Time consistent solutions typically result in greater output levels in the steady state than are obtained when the monopolist can commit, because the seller behaves more aggressively than in the full commitment solution.

In the present context of a monopolist selling in segmented markets, the monopolist will face a time consistency problem in each market. Thus, we would anticipate that steady state stocks will be higher than those under the solution with commitment. However, it is not clear a priori how this will affect relative prices of used goods in the two markets in the steady state. Therefore, it is of interest to find out whether the inability to commit affects the desirability of trade in used goods.

Time consistency only modifies the nature of the solution and therefore, the firm's per-period profit function is still given by equation (4). We solve for the Markov Perfect Equilibrium, where production is only a function of the stock of used goods and which is the unique solution that is recovered by backwards induction. Since the home and foreign markets are independent in the case of a quota, we can solve the problem for the home market alone. The foreign market solution follows by symmetry. The Markov perfect equilibrium is given by a value function $V(\cdot, \cdot)$ and decision rules $g(\cdot)$ that solves

$$V(x_{t-1}) = (p^N(x_t, x_{t-1}, g(x_t)) - c)x_t + \beta V(x_t),$$

where $x_t = g(x_{t-1})$

$$g(x_{t-1}) = \arg\max_{x_t} (p^N(x_t, x_{t-1}, g(x_t)) - c)x_t + \beta V(x_t, x_t^*)$$

Using the envelope theorem, we obtain that $\frac{\partial V(x)}{\partial x} = -bqx$, which yields a solution that is linear in the stock of used goods.

This solution for the firm's decision rule will have the form $g(x) = a_0 + a_1x$. We can solve for these coefficients from the first order conditions, which yields

$$a_1 = -\frac{q}{2 + (2 + 3a_1)\beta q)},\tag{19}$$

$$(20)$$

$$a_0 = \frac{b - c + b\beta q - 2a_0 b\beta q}{b(2 + (2 + 3a_1)\beta q)} - \frac{m(bq + b\beta q)}{b(2 + (2 + 3a_1)\beta q))}$$
(21)

The resulting steady state, given by $a_0/(1-a_1)$ is stable as $a_1 \in [-\frac{q}{2}, 0]$.

Using (19) to solve for the coefficients and then substituting them in the inverse demand functions in (3), we obtain the steady state production level

$$x^{SST} = \frac{3(b(1+\beta q) - c - mbq(1+\beta))}{b(5 + (3+11\beta)q + \sqrt{1+\beta(2-3q)q + \beta^2 q^2})},$$
(22)

The comparison of production between the two solutions at autarky follows from the durable goods literature as expected. Comparing (6) with (22) yields

$$\frac{x^{SST} - x^{SS}}{x^{SS}} = \left(\frac{1 + q(3+\beta) - \sqrt{1 + 2q\beta + q^2(\beta - 3)\beta}}{b(5 + (3+11\beta)q + \sqrt{1 + \beta(2-3q)q + \beta^2q^2})}\right) > 0$$

A monopolist choosing output tomorrow does not internalize how its choice negatively affects past demand by lowering the good's resale price when there is not commitment, and hence overproduces. As a result of the greater output level, steady state prices of both new and used goods will be lower than in the case with commitment.

$$p_N^{SS} = \frac{3c(1+q+2\beta q)+b(1+\beta(1-m)q-mq)(2+5\beta q+\sqrt{1+\beta(2-3q)q+\beta^2q^2})}{5+(3+11\beta)q+\sqrt{1+\beta(2-3q)q+\beta^2q^2}}$$
$$p_U^{SS} = bq\left(\frac{(6q((1+\beta)+bm(1+\beta)))}{5+(3+11\beta)q+\sqrt{1+\beta(2-3q)q+\beta^2q^2}}-1-m\right)$$

Price of both new and used goods will be decreasing in the quantity of imports. Similar expression are obtained for the steady state solution in the foreign country,

$$\begin{split} x^{SS*} &= \frac{3(b^*(1+\beta q^*)-c+mb^*q^*(1+\beta))}{b^*(5+(3+11\beta)q^*+\sqrt{1+\beta(2-3q^*)q^*+\beta^2(q^*)^2})} \\ p_N^{SS*} &= \frac{3c(1+q^*+2dq^*)+b^*(1+\beta(1+m)q^*+mq^*)(2+5\beta q^*+\sqrt{1+\beta(2-3q^*)q^*+\beta^2(q^*)^2})}{5+(3+11\beta)q^*+\sqrt{1+\beta(2-3q^*)q^*+\beta^2(q^*)^2}} \\ p_U^{SS*} &= b^*q^* \left(\frac{(6q^*((1+\beta)-b^*m(1+\beta)))}{5+(3+11\beta)q^*+\sqrt{1+\beta(2-3q^*)q^*+\beta^2(q^*)^2}} - 1 + m\right). \end{split}$$

An increase in home imports reaises the foreign output in the steady state, Foreign prices are increasing in the quantity of home imports, since larger home imports will reduce the quantity of used goods in the foreign market.

The solutions for new and used goods prices for the time consistent case can be used to derive loci in (b^*, q^*) space at which prices of the respective goods are equalized, as was done in Figure 1 for the case with commitment. The expressions for new and used goods prices are sufficiently complex that general statements about the effect of time consistency on these loci trade pattern do not seem possible, so we provide analysis based on the same set of parameter values that have been used in previous Figures. The N and U loci in Figure 4 show the foreign parameter values at which prices of new and used goods are equalized in the case with commitment, as shown in Figure 1. The N^T and U^T loci identify parameter values at which new and used goods prices are equalized in the time consistent case. In each case the parameter values above the loci reflect values for which the prices of goods in the foreign country are higher than at home. For each problem, the locus of values that equalize used goods prices is steeper than that for which new goods prices are equalized, reflecting the larger effect of q^* on used prices. It can be seen that the effects of time consistency on these loci are relatively small. Interestingly, the inability of the monopolist to commit to future output levels has the effect of rotating the locus at which new goods



Figure 2: A Comparison of Prices and Profits with Commitment and Time Consistency

price are equalized upwards, and rotates the locus at which used prices are equalized downward. Thus, the effect of time consistency is to expand the region of parameter values for which the country with the higher price of new goods has a lower price of used goods.

We can also examine the range of parameter values for which trade liberalization will raise the profits of the monopolist. The PP locus identifies the parameter values for which a small increase in imports to the home country will have no effect on the profits of the monopolist in the case with commitment, and P^T identifies the corresponding values for the time consistent solution. For values above (below) the respective loci, an increase in imports by the home country raises (reduces) the profits of the monopolist. Again we find a relatively small difference between the commitment and time consistent solutions. The profits of the monopolist will increase in the region where either the profits of the monopolist are increasing in m and autarky prices of used goods are higher in the home country, or where the profits of the monopolist are decreasing in m and the price of used goods are lower in the home country. This corresponds to the region between the U^T and P^T loci in Figure 4. This region is relatively larger in the case with time consistent production for these parameter values.

Figure 5 also depicts the effect of a small increase in m on worldwide



Figure 3: Welfare Effects of LIberalization under Commitment and Time Consistency

welfare. Parameter values above the W^T curve result in welfare decreasing when m is increased, while parameter values below it, result in an increase in welfare from raising m. Therefore, in the area above U^T , welfare increases if used goods flow to the foreign country. In the area between U^T and W^T , welfare decreases with m as it is still above the W^T curve, but goods would flow from the foreign to the home country, making autarky the preferred option. Lastly, in the area below W^T , welfare increases with m and m > 0because we are below the U^T locus. Thus, welfare increases by increasing m, letting used goods flow from the foreign to the home country. Qualitatively our results match the ones obtained for full commitment.

Figure 5 also plots W, which is the the locus of values for which an increase in m has no effect on welfare under commitment. This curve is below the W^T curve. This create a region where welfare would increase with m with full commitment yet decrease with it with time consistency.

4 Conclusions

In this paper, we have studied the impact of restrictions on trade in used goods between countries when the seller of new goods is a monopolist and sells in markets in both countries. We provide a characterize of the effect of trade liberalization on profits and welfare. Two findings are that the monopolist seller may prefer used goods markets to be liberalized and aggregate welfare may be maximized when markets are autarkic. These are in sharp contrast with the common wisdom that manufacturers benefit from protectionism and welfare increases with liberalization. We know that the liberalization of used goods markets has typically been slow when compared to the liberalization of markets for other goods, which could suggest that the welfare gains from liberalizing trade in used goods are unclear. The one clear implication that stems from our results is that agreements towards liberalizing used markets should be carefully assessed. Our analysis, however, stands mute on the environmental implications.

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Appendix

Derivation of Slopes of Loci for Figure 1:

The NN is the locus of values of (b^*, q^*) that keep p_N^{SS*} constant and equal to p_N^{SS} . Similarly, the UU locus holds p_U^{SS*} constant at p_U^{SS} Differentiation of (8) yields

$$\frac{db^*}{dq^*}\Big|_{NN} = -\frac{b^*\beta}{1+q^*\beta} \qquad \frac{db^*}{dq^*}\Big|_{NN} = -\frac{c+b^*q^*\beta(1+\beta)(2+q^*+2q^*\beta)}{q^{*2}(1+\beta)(2+q^*+2q^*\beta)}$$

and

$$\left. \frac{db^*}{dq^*} \right|_{NN} - \left. \frac{db^*}{dq^*} \right|_{NN} = \frac{c + b^* q^* \beta (1+\beta) (2+q^*+2q^*\beta)}{q^{*2} (1+\beta) \left(1+q^*\beta\right) \left(1+q^*+2q^*\beta\right)} > 0$$

Both curves go through the point (b, q), so this inequality establishes that the UU locus will be steeper at the intersection point. Since this relationship of relative slopes must hold at any intersection point and both loci are negatively sloped, the loci must have a single intersection point.

The locus of values at which $\frac{d\pi^{SS}}{dm} + \frac{d\pi^{SS*}}{dm}\Big|_{m=0} = 0$ must also go through the point (b,q) in Figure 1. Differentiating (9) yields

$$\frac{db^*}{dq^*}\Big|_{\Pi^W} = -\frac{b^* \left(1 + 2q^*\beta + q^{*2}\beta(1+2\beta)\right) - c}{q^* \left(1 + q^*\beta\right) \left(1 + q^* + 2q^*\beta\right)} < 0$$

The numerator of this expression will be positive for all parameter values satisfying $x^{*SS} > 0$ at autarky, which requires $b^*(1 + q^*\beta) > c$. Comparing the slope of this locus with the NN and UU locus establishes that it must lie between the two loci:

$$\frac{db^*}{dq^*}\Big|_{\Pi^W} - \frac{db^*}{dq^*}\Big|_{NN} = -\frac{2b^*x^{*SS}}{q^*(1+q^*\beta)} < 0$$

$$\frac{db^*}{dq^*}\Big|_{\Pi^W} - \frac{db^*}{dq^*}\Big|_{NN} = \frac{c+b^*q^*(1+\beta)}{q^{*2}(1+\beta)(1+q^*\beta)} > 0$$