EFIGE IS A PROJECT DESIGNED TO HELP IDENTIFY THE INTERNAL POLICIES NEEDED TO IMPROVE EUROPE'S EXTERNAL COMPETITIVENESS

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Trade, innovation and productivity: a quantitative analysis of Europe

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European Firms In a Global Economy EFIGE

SEVENTH FRAMEWORK PROGRAMME

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Trade, Innovation and Productivity:
A Quantitative Analysis of Europe*

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Abstract

This paper proposes a trade model with heterogeneous firms that decide not just whether and how much to export but also whether and how much to innovate. Incorporating both the extensive and intensive margins of trade and innovation leads to different possible equilibria. Depending on how costly trade is relative to innovation, medium-productivity firms may either export without innovating, innovate without exporting, do both or do neither. The impact of trade on aggregate productivity and welfare depends crucially on the equilibrium the economy is in. When lowering the variable costs of trade, the welfare effects arising from reallocating market shares across firms may be non-negligible, and when lowering the fixed cost of trade, aggregate productivity need not always increase. After calibrating the model to five European countries, we show that the different equilibria are plausible, and provide quantitative evidence that supports the predictions of our theory.

JEL Codes: F12, F14, O24, O31
Keywords: Process Innovation, Firm Heterogeneity, Trade Policy

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1 Introduction

There is substantial heterogeneity across firms in both innovation and export activities. Some firms neither innovate nor export, others both innovate and exports, and still others may do one of the activities without the other. In addition, within these different groups of firms, the intensity of both activities also differs across firms. While the literature has long recognized the interdependence between innovation and trade, it has so far not analyzed the impact of trade liberalization on productivity and welfare in a model that incorporates both the extensive and the intensive margins of both trade and innovation.

The main point of the paper is to show that introducing these different margins is key for understanding the impact of trade liberalization. Different equilibria may arise, depending on the relative costs of trade and innovation. After theoretically discussing the properties of each of those equilibria, we show that they are quantitatively plausible by calibrating the model to five European countries. I then show that the impact of trade liberalization depends crucially on the equilibrium the economy is in and the nature of the liberalization. For example, in the case of a drop in variable trade costs, this paper shows that the effects on welfare from changes in firms' decisions to export and innovate may be non-negligible, in contrast to the literature.[1] As another example, a drop in the fixed cost of trade need not always have a positive effect on aggregate productivity. Indeed, in an economy in which many firms export, but few firms innovate, lowering the fixed cost of trade, by increasing the number of exporters, may make innovating more expensive, thus lowering aggregate productivity.

The paper proposes a trade model with heterogeneous firms in the spirit of Melitz (2003) with a basic difference: once a firm learns about its productivity, it can decide to spend resources on innovation to lower its marginal costs. Innovation is a costly activity that involves both fixed and variable costs, hence firms decide not only whether to innovate but also how much to innovate. This is key to be able to explore how trade liberalization affects the extensive and intensive margin of innovation. The model is rich enough to explore the interdependence between the innovation and the export decisions, and yet tractable enough to aggregate up from firm level decisions and analyze how aggregate productivity and welfare respond to changes in trade and innovation policies.

Three different equilibria may arise, depending on how costly trade is relative to innovation. In all three equilibria, high-productivity firms always export and innovate, while low-

[1] See Arkolakis et al. (2012) and Atkeson and Burstein (2010) on this topic.

productivity firms never export or innovate. What differs across equilibria is the behavior of medium-productivity firms. In the *low cost innovation equilibrium*, trade is relatively costly compared to innovation, so that medium-productivity firms innovate, but do not export. In the *low cost trade equilibrium*, trade costs are relatively low compared to innovation, so that medium-productivity firms export, but do not innovate. In between these two extremes, there is the *intermediate equilibrium*, characterized by medium-productivity firms engaging in either both activities or none of them. Depending on which equilibrium the economy is in, the theory illustrates that the effect of trade liberalization on aggregate productivity and welfare may be very different.

To assess the plausibility of the theory, we calibrate the model to five European countries. In particular, the model is calibrated to match a number of salient features of innovation, firm size distribution and international trade in France, Germany, Italy, Spain and United Kingdom, using the firm-level data set *European Firms in a Global Economy* (EFIGE). The survey, conducted during the year 2009, is representative of the manufacturing sector in each country. Especially relevant for our analysis is the information on employment, internationalization and innovation. A first result is that the different equilibria are not only theoretically relevant, but also empirically plausible: different countries are in different equilibria. This is important, since the theory predicts that the effect of trade liberalization on aggregate productivity and welfare depends crucially on the equilibrium a country is in.

A first quantitative exercise consists of quantifying the effect of a reduction in variable trade costs on aggregate productivity. The analysis is based on the *ideal measure* of aggregate productivity defined by Atkeson and Burstein (2010). I focus on this measure, because it captures the productivity that is relevant for welfare. Apart from the direct cost savings effects of a drop in variable trade costs, the theory predicts that there are a number of indirect effects. First, it induces the exit of less productive firms and the reallocation of market shares towards the more productive firms. This is the selection effect described in Melitz (2003). Second, the innovation intensity increases with the participation in foreign markets, so the effect through the intensive margin of innovation should be positive[2]. Third, the theory predicts that the effect through the extensive margin of innovation can be positive or negative. In the *low cost trade equilibrium* and the *intermediate equilibrium*,

[2]Despite the intensity of innovation from domestic firms decreasing (if there are in the economy), the increase on the intensity of innovation of exporter firms ensures that the final effect through the intensive margin is positive.

all innovators are exporting. In that case a decrease in variable trade costs increases the incentives to be an exporter (and to be an exporter innovator), so that the effect through the extensive margin of innovation is positive. In contrast, in the *low cost innovation equilibrium*, some of the innovators do not export. In that case, a drop in trade costs makes it harder for domestic firms to innovate, so that the effect through the extensive margin of innovation is negative.

My findings corroborate the theoretical predictions. In particular, in most countries the effect of a drop in variable trade costs on aggregate productivity through the extensive margin is positive, except in those that are in the *low cost innovation equilibrium*, where the effect is negative. My findings also shed new light on which channels matter when analyzing the impact of trade liberalization on aggregate productivity. Work by Atkeson and Burstein (2010) has suggested that the indirect effects of trade liberalization on productivity are negligible. That is, liberalizing trade improves productivity through the standard direct effect of saving resources on trade, whereas the indirect effects coming from changes in firms' decisions related to exit, trade and innovation are essentially zero. In contrast, our findings show that this depends crucially on the equilibrium an economy is in. While in most countries the indirect effects are indeed negligible, this is not the case of countries in the *low cost innovation equilibrium*. This underscores the importance of having a model that encompasses both the extensive and intensive margins of trade and innovation.

A second quantitative exercise focuses on the effectiveness to increase productivity of lowering the fixed costs of trade or innovation. While our first exercises focused on a reduction in variable trade costs, we now show that a reduction in fixed trade or innovation costs may also have very different effects, depending on the equilibrium the economy is in. While in general the effect of lowering the fixed cost of trade is positive, we find that in the *low cost trade equilibrium* it is negative. The intuition is as follows. In such equilibrium, there are many exporters, but only the most productive innovate. Since all innovators are also exporters, by increasing the incentives to enter the export market, a drop in the fixed costs of trade pushes up real wages, reducing the incentives to innovate. As a result, both the number of innovators and the intensity of the remaining innovators decline, which translates in the final effect on welfare being negative.

The simulations reveal that a non-infinitesimal drop in fixed trade costs, can induce productivity gains from 1% to 20% in total, and only if the economy is already very open (in the *low cost trade equilibrium*) might a further drop in fixed trade costs be damaging to the economy, which suggest that a fixed trade cost liberalization does not have the same nature than a variable trade cost liberalization. In contrast to a fixed trade cost reduction, a fixed innovation cost drop has little effect on the productivity, the maximum increase being around 2%, and has far more damaging effects if it induces economies to be less export oriented, since then the productivity might decrease by up to 7%.

This paper is related to different strands of the literature. On the one hand, there is the literature that focuses on how firms make joint decisions on exporting and innovating. Yeaple (2005) and Bustos (2011) consider models in which there is a binary technology choice, and highlight how firms decide to both enter the export market and adopt the new technology. The cost of innovation is therefore modeled as a fixed cost. Costantini and Melitz (2008) extend this type of joint decision to a dynamic framework where firms face both idiosyncratic uncertainty and sunk costs for both exporting and technology adoption. On the other hand, there is the literature that focuses on examining the impact of trade on the intensity of innovation. Vannoorenberghe (2008) and Rubini (2011) consider models in which firm productivity is endogenously determined through innovation, and highlight that innovation is affected by the existence of foreign markets. Closely related to these is the work of Atkeson and Burstein (2010). They propose a dynamic trade model to include a process innovation decision by incumbent firms following Griliches (1979)'s model of knowledge capital.

A key contribution of my work is joining the two branches of the literature on trade and innovation. While my model abstracts from the dynamics, it explores quantitatively the responses of firms along both the extensive and intensive margins of innovation to changes in the environment. My results echoe those of Atkeson and Burstein (2010) in that welfare gains from trade do not depend on how a change in variable trade costs affects firms' exit, export and innovation decisions, if the extensive margin of innovation is not affected by the policy. At the same time, my result complements theirs by explaining carefully how a negative incentive to innovate, driven by a drop in variable trade costs, actually implies that firms' exit, export and innovation decisions can have an impact on welfare gains.

Finally, my work here is also related to a large literature on the aggregate implications of trade liberalization. Baldwin and Robert-Nicoud (2008) study a variant of Melitz's model that features endogenous growth through spillovers. They show that depending on the nature of the spillovers, a reduction in international trade costs can increase or decrease growth through changes in product innovation. My model centers on process innovation and abstracts from such spillovers. Arkolakis et al. (2012) calculate the welfare gains from trade in a wide class of trade models, including Krugman (1980) and Melitz (2003) models with Pareto productivities. The main differences between this paper and mine is that they abstract from innovation and focus only on changes in marginal trade costs.

The paper is organized as follows. In Section 2, I present the model of the economy where firms take decisions on innovation and exporting. In Section 3, I explore the equilibria determined by the interaction between the exporting and innovation choices creates. In Section 4, I calibrate the model to match five main European economies. In Section 5, I analyze the effects in aggregate productivity and welfare of a drop in variable trade costs, a drop in fixed trade cost and a drop in fixed innovation costs. Section 6 concludes.

2 Model

The model is based on the monopolistic competition framework proposed by Melitz (2003). I consider a symmetric $n+1$ country world, each of which uses a single factor of production (labor L) to produce goods. In contrast to Melitz (2003), the model allows these firms to have the opportunity to engage in process innovation.

2.1 Demand

I denote the source country by i and the destination country by j, where $i, j = 1, ..., n+1$. In each country j, there is a continuum of consumers of measure L_j. Given the set Ω of varieties supplied to the market, the consumer's preferences of country j are represented by the standard C.E.S. utility function

$$\left[\int_{\omega\in\Omega} q_{ij}^{\rho}(\omega)d\omega\right]^{\frac{1}{\rho}}$$

where $q_{ij}(\omega)$ denotes the quantity consumed of variety ω produced by firm i in country j and $\sigma = \frac{1}{1-\rho} > 1$ is the elasticity of substitution across varieties. The market is subject to the expenditure-income constraint:

$$\int_{\omega\in\Omega} p_{ij}(\omega)q_{ij}(\omega)d\omega = R_j$$

where R_j is the total revenues obtained in country j.

Then standard utility maximization implies that the demand for each individual variety will be:

$$q_{ij}(\omega) = [p_{ij}(\omega)]^{-\sigma} \frac{R_j}{P_j^{1-\sigma}} \tag{1}$$

where $p_{ij}(\omega)$ is the price of each variety ω and $P_j = \left[\int_{\omega \in \Omega} p_{ij}(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$ denotes the price index of the economy.

2.2 Supply

There is a continuum of firms, each producing a different variety ω. Each firm draws its productivity φ from a distribution $G(\varphi)$ with support $(0, \infty)$ after paying a labor sunk cost of entry f_E. Since a firm is characterized by its productivity φ, it is equivalent to talk about variety ω or productivity φ.

Production requires only labor, which is inelastically supplied at its aggregate level L_j, and therefore can be taken as an index of country's j size. In contrast to the Melitz model where firms use a constant returns to scale production technology, firms can affect their marginal cost through process innovation. To enter country j, firm i needs $f_{ij} > 0$ labor units and I make the standard iceberg cost assumption that $\tau_{ij} > 1$ units of the good have to be produced by firm i to deliver one unit to country j. Without loss of generality, I assume that $\tau_{ii} = 1$ and thus I denote $\tau_{ij} = \tau \; \forall i \neq j$.[3] Therefore, to produce output $q_{ij}(\varphi)$, a firm requires $l_{ij}(\varphi)$ labor units

$$l_{ij}(\varphi) = f_{ij} + c(z(\varphi)) + \frac{q_{ij}(\varphi)}{\varphi} \frac{\tau_{ij}}{(1 + z(\varphi))^{\frac{1}{\sigma-1}}}$$

where $z(\varphi)$ is a measure of the productivity increase from innovation that has an associated cost function $c(z(\varphi))$.

The cost function of the innovation follows Klette and Kortum (2004), Lentz and Mortensen (2008) and Stähler et al. (2007). Firms pay a fixed cost, that can be attributed to the acquisition and implementation of the technology, plus a variable cost that depends directly on the process innovation performed by each firm. Hence the cost function $c(z_i)$ is defined as

$$c(z(\varphi)) = \begin{cases} z(\varphi)^{\alpha+1} + f_I & \text{if } z(\varphi) > 0 \\ 0 & \text{if } z(\varphi) = 0 \end{cases}$$

where f_I is the fixed cost required to implement the process innovation and $\alpha > 0$ measures

[3]Note that $\tau_{ij} = \tau_{ji}$ by symmetry and there is no possibility of transportation arbitrage

the rate at which the marginal cost of the innovation increases. Thus, the higher the level of innovation, the higher the cost associated with marginal increases.

Even though it can be argued that the cost of innovation can be simplified by imposing a linear variable cost, the existence of convex innovation costs is a standard feature in the literature and ensures that innovation is finite. Another simplification would be to have either a fixed cost or a variable cost but not both. Nevertheless maintaining a flexible cost function is important. For example, Vannoorenberghe (2008) assumes away a fixed innovation cost, which implies that all firms engage in process innovation. This eliminates the possibility of studying the interaction between the export and innovation decisions along the extensive margin, which is one of the purposes of this paper.

2.3 Firm's problem

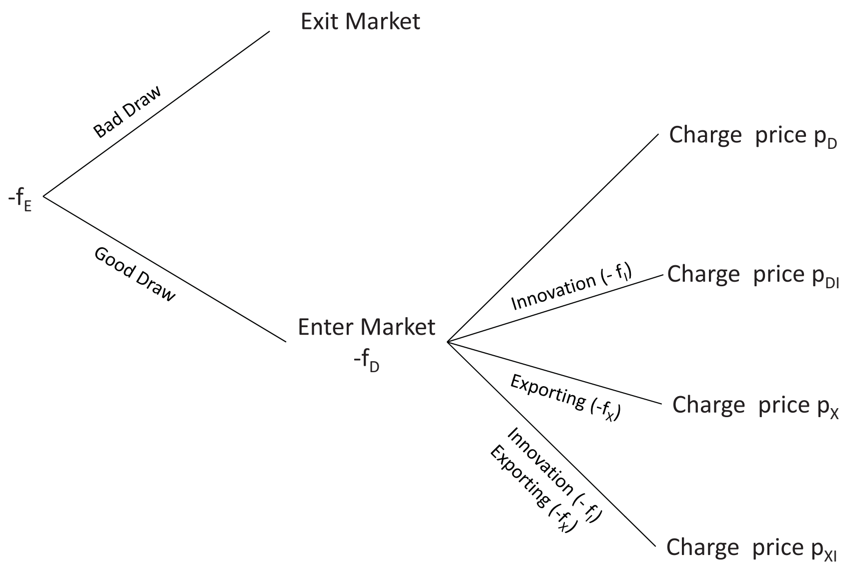

Figure 1: Timing

Figure 1 represents the timing of the firm's problem. In a first stage, as in Melitz (2003), entering the market means paying a labor sunk cost f_E, in order to get a draw of the productivity parameter φ. In the second stage, with the knowledge of their own productivity, firms decide which activities to undertake. Since both exporting and innovation require paying a labor fixed cost, f_X and f_I respectively, there will be four types of firms in the open economy. Type D firms are only active in the domestic market and do not perform innovation; Type DI firms are those active only in the domestic market that innovate; Type X firms are those active in both the domestic and the foreign market that do not perform any innovation; and Type XI firms are active in the domestic and foreign markets that engage in innovation activities. Finally, in the third stage, firms decide prices. Given the timing, I solve the firms problem through backward induction.

Optimal Pricing Rule. In the last stage of the problem the firm sets its optimal price, given its innovation decision and the market conditions which are summarized by the price index P_j and R_j.

$$\max_{p_{ij}(\varphi)} p_{ij}(\varphi)\, q_{ij}(\varphi) - f_{ij} - \frac{\tau_{ij} q_{ij}(\varphi)}{\varphi\left[(1+z_i)^{\frac{1}{\sigma-1}}\right]} - c(z_i)$$

The corresponding first order condition is

$$p_{ij}(\varphi) = \left(\frac{\sigma}{\sigma-1}\right)\frac{\tau_{ij}}{\varphi}\cdot\frac{1}{(1+z_i)^{\frac{1}{\sigma-1}}} \quad \forall\, z \tag{2}$$

Optimal Innovation Decision. The returns of process innovation increase with the participation in more countries. Thus, the optimal innovation rule for firm i is obtained from the first order condition of the maximization of $\sum_j \pi_{ij}(\varphi) = \sum_j [p_{ij}(\varphi)q_{ij}(\varphi) - l_{ij}(\varphi)]$ with respect to z_i, provided that the firm makes higher profits by innovating than by choosing not to innovate. This gives

$$z_i(\varphi) = \begin{cases} \left[1+n\tau^{1-\sigma}\right]^{\frac{1}{\alpha}}\left[\frac{1}{\alpha+1}\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{1}{\alpha}} & \text{if } \sum_j \pi_{ij}^{I}(\varphi) \geq \sum_j \pi_{ij}^{NI}(\varphi) \\ 0 & \text{if } \sum_j \pi_{ij}^{I}(\varphi) < \sum_j \pi_{ij}^{NI}(\varphi) \end{cases} \tag{3}$$

where $\frac{1}{\alpha}$ is the parameter that shapes the optimal innovation function and tells us how innovation rises with size, where I take the productivity parameter $\varphi^{\sigma-1}$ to be the indicator of size. If the function is linear ($\alpha = 1$), then innovation rises proportionately with size, however, if the function is concave ($\alpha > 1$), then the amount of innovation performed will rise less than proportionally with size, and if the function is convex ($0 < \alpha < 1$) the amount of innovation performed will increase more than proportionally with the productivity.

To make the joint decision of whether to enter the foreign markets and whether to innovate or not, firms will choose the option that yields the highest profits. Since countries are symmetric we can drop the subscripts and classify firms in four types.

- Profits of a domestic non-innovator firm (Type D):

$$\pi_D = \frac{R\left(P\rho\right)^{\sigma-1}}{\sigma}\varphi^{\sigma-1} - f_D$$

- Profits of a domestic innovator firm (Type DI):

$$\pi_{DI} = \frac{R\left(P\rho\right)^{\sigma-1}}{\sigma}\varphi^{\sigma-1}\left(1+z_D\left(\varphi\right)\right) - f_D - c\left(z_D\left(\varphi\right)\right)$$

- Profits of an exporter non-innovator firm (Type X):

$$\pi_X = \left(1+n\tau^{1-\sigma}\right)\frac{R\left(P\rho\right)^{\sigma-1}}{\sigma}\varphi^{\sigma-1} - nf_X - f_D$$

- Profits of an exporter innovator firm (Type XI):

$$\pi_{XI} = \left(1+n\tau^{1-\sigma}\right)\frac{R\left(P\rho\right)^{\sigma-1}}{\sigma}\varphi^{\sigma-1}\left(1+z_X\left(\varphi\right)\right) - nf_X - f_D - c\left(z_X\left(\varphi\right)\right)$$

where $f_D = f_{ii}$, $f_X = f_{ij} = f_{ji}\ \forall j \neq i$, $z_D(\varphi) = \left[\frac{1}{\alpha+1}\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{1}{\alpha}}$, and $z_X(\varphi) = \left[1+n\tau^{1-\sigma}\right]^{\frac{1}{\alpha}}\left[\frac{1}{\alpha+1}\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{1}{\alpha}}$.

3 Equilibrium

There will be three different equilibria that will cover the whole parameter space. First, the *low-cost innovation equilibrium*, where the activity of exporting is relatively costly in comparison to innovation, and therefore only the most productive firms will carry out both activities, middle productivity firms will innovate but not export and the lower productivity firms will neither innovate nor export. Second, the *low-cost trade equilibrium,* where the activity of innovation is relatively costly in comparison to exporting and therefore only the most productive firms will carry out both activities, middle productivity firms will export but not engage in innovation and the lower productivity firms will neither innovate nor export. Thirdly, between these two equilibria there will be the *intermediate equilibrium* where firms are either very productive and can undertake both activities or do not perform any of them.

The existence of these three equilibria is consistent with the empirical evidence found both in the trade and the innovation literature. Costantini and Melitz (2008) suggest that exporting and innovation are performed by the most productive firms while domestic producers are typically less innovative and less productive, a feature common to all the equilibria. Vives (2008) provides intuition for the decisions taken by middle productivity firms in each equilibrium. If trade costs are relatively high, middle productivity firms are domestic innovators because being an exporter without innovating is not profitable. A decrease in trade costs attracts the most productive firms from the foreign country, discouraging middle productivity domestic firms to undertake innovation. The disappearance of domestic innovators as trade costs fall can be explained by this Schumpeterian effect and is also predicted by the dynamic model of Costantini and Melitz (2008). However, a fall in trade costs enables more firms to participate actively in both markets which explains the existence of exporter non-innovators when trade costs are low enough.

Different theoretical papers have identified these equilibria separately, but never all in a single model. Bustos (2011) identifies the equilibrium where there are no domestic innovators firms since it is an unprofitable choice. In Vannoorenberghe (2008) all firms innovate, therefore it is not possible to study the interaction between both decisions. Finally, Navas-Ruiz and Sala (2007) identify the two extreme equilibria, but fail to identify the intermediate equilibrium. The main contribution of the theoretical model is the identification of all the equilibria with the ability to study the transitions between them and the possible productivity gains that might occur through the intensive and extensive margins of innovation. In the numerical section I will analyze whether these different equilibria are relevant when calibrating the model to different European countries. In what follows I describe each of the equilibria, the effects that trade has on innovation in each case, the parameter restrictions that give rise to the different equilibria, and conclude by focusing on the interaction between exporting and innovation.

3.1 Low Cost Innovation Equilibrium

The *low cost innovation equilibrium* is characterized by exporting being less attractive than innovation. In Figure 2, I depict the profits of all types of firms as a function of productivity when trade costs are relatively high in comparison to innovation costs. The envelope line shows the type of firm that will be chosen by a firm with productivity φ as it maximizes profits. In this equilibrium, the least productive firms ($\varphi < \varphi_D$) exit, the low productivity firms ($\varphi_D < \varphi < \varphi_{DI}$) are active in the domestic market but do not innovate or export, middle productivity firms ($\varphi_{DI} < \varphi < \varphi_{XI}$) are active only on the domestic market but innovate, and the most productive firms ($\varphi > \varphi_{XI}$) are active both in the domestic and export market, and innovate. Note that there is no range of productivity level where exporting without innovating is profitable, that is, the marginal exporter is an innovator as well.

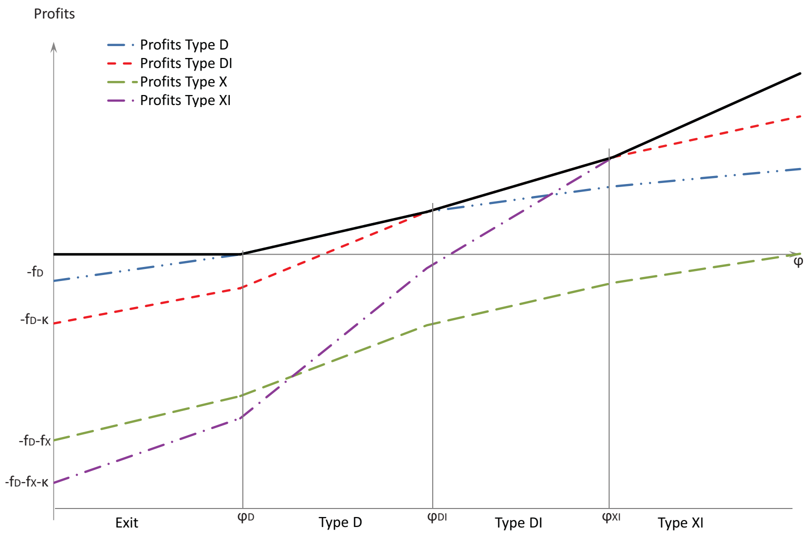

Figure 2: Low Cost Innovation Selection Path

The conditions of entry in the domestic and export markets plus the innovation condition allow to solve for the different productivity cutoffs in the *low cost innovation equilibrium.*

The Zero Profit Condition (ZPC) in the domestic market is $\pi_D(\varphi_D^*) = 0$, so that:

$$(\varphi_D^*)^{\sigma-1} = \frac{f_D}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)} \tag{4}$$

The Innovation Profit Condition (IPC) determines the productivity cutoff φ_{DI}^* which is the productivity of the firm indifferent between innovating or not while operating only on the domestic market, i.e. $\pi_{DI}(\varphi_{DI}^*) = \pi_D(\varphi_{DI}^*)$, so that:

$$(\varphi_{DI}^{*})^{\sigma-1} = \frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)} \tag{5}$$

The Innovation Export Profit Condition (IXPC) determines the exporting-innovation cutoff φ_{XI}^{*} which is the productivity of an innovating firm indifferent between participating also on the exporting market or not:

$$\pi_{XI}(\varphi_{XI}) - \pi_{DI}(\varphi_{XI}) = 0 \tag{6}$$

The following proposition shows for which part of the parameter space the *low cost innovation equilibrium* exists.

Proposition 1.

The economy is in the low cost innovation equilibrium, $\varphi_{XI}^{*} > \varphi_{DI}^{*} > \varphi_{D}^{*}$*, if the following parameter restrictions hold*

1. $\tau^{\sigma-1} f_X \geq \frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right]}{n\tau^{1-\sigma}} f_I + \left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)$
2. $\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1) \geq f_D$

Proof. The formal proof can be found in the Appendix A. The proof is divided in two parts. First I show that there exist a single solution to equation (6). The non linearity present in the optimal innovation decision is the source of the complexity of finding a closed form for the cutoff φ_{XI}^{*}. Nevertheless, I show that selection into exporting and innovation $(\varphi_{XI}^{*} > \varphi_{DI}^{*})$ requires that condition 1 of Proposition 1 holds, that is exporting costs should be high enough relative to innovation costs. Notice that condition 2 of Proposition 1 ensures that there is selection into innovation $(\varphi_{DI}^{*} > \varphi_{D}^{*})$. Secondly, I show that equations (4) to (6) along with the Free Entry (FE) condition, which requires that the sunk entry cost equals the present value of expected profits:

$$\frac{1}{\delta}\left[\int_{\varphi_{D}^{*}}^{\varphi_{DI}^{*}} \pi_D(\varphi)\, dG(\varphi) + \int_{\varphi_{DI}^{*}}^{\varphi_{XI}^{*}} \pi_{DI}(\varphi)\, dG(\varphi) + \int_{\varphi_{XI}^{*}}^{\infty} \pi_{XI}(\varphi)\, dG(\varphi)\right] = f_E \tag{7}$$

uniquely determine the equilibrium price (P), the number of firms (M) and the distribution of active firms productivity in the economy along with the productivity cutoffs φ_{D}^{*}, φ_{DI}^{*} *and* φ_{XI}^{*}. □

3.2 Low Cost Trade Equilibrium

The *low cost trade equilibrium* is characterized by exporting being more attractive than innovation. In Figure 3 , I depict the profits of all types of firms as a function of productivity when trade costs are relatively low in comparison to innovation costs. The envelope line shows the type of firm that will be chosen by a firm with productivity φ as it maximizes profits. In this equilibrium, the least productive firms ($\varphi < \varphi_D$) exit, the low productivity firms ($\varphi_D < \varphi < \varphi_{DI}$) are active in the domestic market but do not innovate or export, middle productivity firms ($\varphi_{DI} < \varphi < \varphi_{XI}$) are active only on the domestic market but innovate, and the most productive firms ($\varphi > \varphi_{XI}$) are active both in the domestic and export market, and innovate. Note that there is no range of productivity level where innovation without exporting is profitable, that is, the marginal innovator is an exporter.

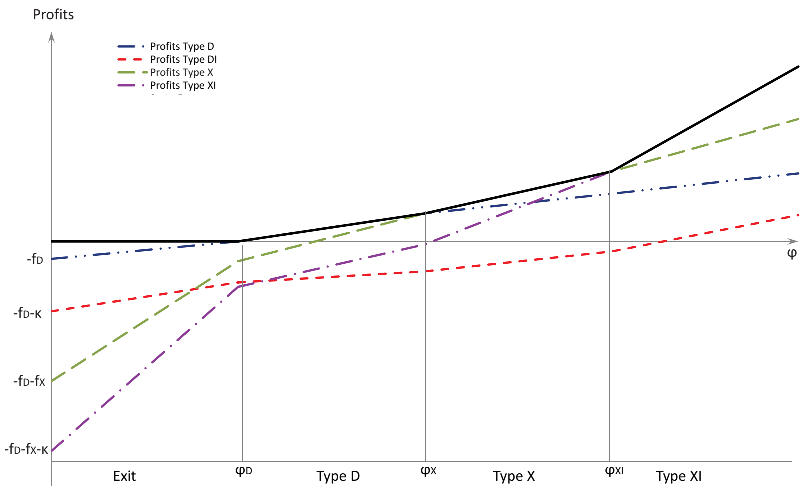

Figure 3: Low Cost Trade Selection Path

The conditions of entry in the domestic and export markets, plus the innovation conditions, allow to solve the different productivity cutoffs in the *low cost trade equilibrium.*

The Zero Profit Condition (ZPC) in the domestic market[4] is $\pi_D(\varphi_D^*) = 0$ so that:

$$(\varphi_D^*)^{\sigma-1} = \frac{f_D}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)} \tag{8}$$

[4]The ZPC condition is defined theoretically in the same way in every equilibrium. However, since the aggregates in each situation are different, the entry cutoff will also be different.

The Exporting Profit Condition (XPC) determines the exporting-entry productivity cutoff φ_X^* which is the productivity of the firm indifferent between staying in the domestic market and participating in the export market, i.e. $\pi_X(\varphi_X^*) = \pi_D(\varphi_X^*)$:

$$(\varphi_X^*)^{\sigma-1} = \frac{f_X}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\tau^{1-\sigma}} \tag{9}$$

The Exporting Innovation Profit Condition (XIPC) determines the innovation exporting productivity cutoff φ_{XI}^*, which is the productivity of an exporting firm indifferent between innovating or not, i.e. $\pi_{XI}(\varphi_{XI}^*) = \pi_X(\varphi_{XI}^*)$:

$$(\varphi_{XI}^*)^{\sigma-1} = \frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)(1+n\tau^{1-\sigma})} \tag{10}$$

The following proposition shows for which part of the parameter space the *low cost trade equilibrium* exists.

Proposition 2.

The economy is in the low cost trade equilibrium, $\varphi_{XI}^* > \varphi_X^* > \varphi_D^*$, if the following parameter restrictions hold

$$\frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{(1+n\tau^{1-\sigma})} \geq \tau^{\sigma-1} f_X \geq f_D$$

Proof. Selection into exporting and innovation ($\varphi_{XI}^* > \varphi_X^*$) requires innovation costs to be high enough relative to trade costs and selection into exporting ($\varphi_X^* > \varphi_D^*$) requires trade costs to be high enough relative to production costs. Equations (8) *to* (10) along with the Free Entry (FE) condition, which requires that the sunk entry cost equals the present value of expected profits:

$$\frac{1}{\delta}\left[\int_{\varphi_D^*}^{\varphi_X^*} \pi_D(\varphi)\, dG(\varphi) + \int_{\varphi_X^*}^{\varphi_{XI}^*} \pi_X(\varphi)\, dG(\varphi) + \int_{\varphi_{XI}^*}^{\infty} \pi_{XI}(\varphi)\, dG(\varphi)\right] = f_E \tag{11}$$

uniquely determine the equilibrium price (P), the number of firms (M) and the distribution of active firms productivity in the economy along with the productivity cutoffs φ_{XI}^*, φ_X^* *and* φ_D^*. See Appendix B for a formal proof. □

3.3 Intermediate Equilibrium

The *intermediate equilibrium* is characterized by exporting and innovation being relatively equally attractive. In Figure 4, I depict the profits of all types of firms as a function of productivity when trade costs are neither very high nor very low in comparison to innovation costs. The envelope line shows the type of firm that will be chosen by a firm with productivity φ as it maximizes profits. In this equilibrium, the least productive firms ($\varphi < \varphi_D$) exit, the low productivity firms ($\varphi_D < \varphi < \varphi_{XI}$) are active in the domestic market but do not innovate or export, and the most productive firms ($\varphi > \varphi_{XI}$) are active both in the domestic and export market, and innovate. Note that there is no range of productivity level where exporting without innovating or innovating without exporting is profitable, that is, the marginal exporter is an innovator as well.

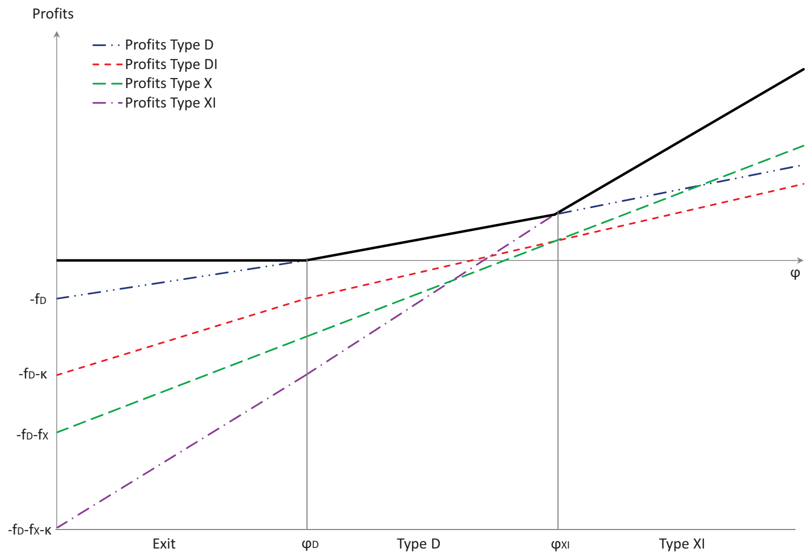

Figure 4: Intermediate Selection Path

The conditions of entry in the domestic markets, plus the innovation and export condition, allow to solve the different productivity cutoffs in the *intermediate equilibrium.*

The Zero Profit Condition (ZPC) in the domestic market[5] is $\pi_D\left(\varphi_D^*\right) = 0$ so that:

$$\left(\varphi_D^*\right)^{\sigma-1} = \frac{f_D}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)} \tag{12}$$

The Exporting Innovation Profit Condition (XIPC) determines the innovation exporting productivity cutoff φ_{XI}^*, which is the productivity of a firm indifferent between exporting and innovating or not.

$$\pi_{XI}\left(\varphi_{XI}^*\right) - \pi_D\left(\varphi_{XI}^*\right) = 0 \tag{13}$$

The following proposition shows for which part of the parameter space the *intermediate equilibrium* exists.

Proposition 3.

The economy is in the intermediate equilibrium, $\varphi_{XI}^* > \varphi_D^*$, if the following parameter restrictions hold

1. $\frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]}{n\tau^{1-\sigma}} f_I + \left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1) \geq \tau^{\sigma-1} f_X$

2. $\tau^{\sigma-1} f_X \geq \frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{(1+n\tau^{1-\sigma})}$

3. $\frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{(1+n\tau^{1-\sigma})} \geq f_D$

Proof. If the first parameter restriction does not hold, then for some firms is profitable to innovate without exporting. If the second parameter restriction does not hold, then for some firms is profitable to export without innovating. Therefore, the trade costs must be in between the limits of innovation, so that firms either export and innovate or simply remain in the domestic market. The non linearity present in the optimal innovation decision is the source of the complexity of finding a closed form for the cutoff φ_{XI}^*, nevertheless I show that conditions 1 and 2 hold. Furthermore, I show that Equations (12) *and* (13) along with the Free Entry (FE) condition, which requires that the sunk entry cost equals the

[5]The ZPC condition is defined theoretically in the same way in every equilibrium. However, since the aggregates in each situation are different, the entry cutoff will also be different.

present value of expected profits:

$$\frac{1}{\delta}\left[\int_{\varphi_D^*}^{\varphi_{XI}^*}\pi_D\left(\varphi\right)dG\left(\varphi\right)+\int_{\varphi_{XI}^*}^{\infty}\pi_{XI}\left(\varphi\right)dG\left(\varphi\right)\right]=f_E \tag{14}$$

uniquely determine the equilibrium price (P), the number of firms (M) and the distribution of active firms productivity in the economy along with the productivity cutoffs φ_{XI}^* *and* φ_D^*. See Appendix C for a formal proof. □

3.4 Discussion

The firm productivity distribution varies along the parameter space according to the relation between trade costs and the relative innovation costs. This is especially relevant for firms with an intermediate level of productivity, as their decisions will be most sensitive to these costs. In particular, in the *low cost innovation equilibrium*, when trade costs are high enough, they are domestic innovators. In the *low cost trade equilibrium*, when trade costs are low enough in relation to innovation costs, middle productivity firms will be exporters and the most productive of them will export and innovate. In between these two equilibria, there is the *intermediate equilibrium*, where trade costs are not relatively high enough for firms to be domestic innovators nor low enough for firms to be exporters non-innovators. That is, middle productivity firms are either exporter innovators or domestic firms. These choices are the ones that determine the parameter restrictions associated to each equilibrium. Furthermore, notice that the three equilibria cover the whole parameter space, and therefore the firm productivity distribution and the effects of opening up to trade of an economy can be always determined. Table 1 summarizes all the possible equilibria in the open economy and the parameter restrictions associated to each one.

Furthermore, the model has implications for the aggregate productivity level. Firstly, trade induces the exit of the less productive firms and the reallocation of market shares towards the more productive firms, raising the industry average productivity in the long run. This is the selection effect described in Melitz (2003). And secondly, trade has indirect effects on the average productivity through innovation. Moving from the *low cost innovation equilibrium* to the *low cost trade equilibrium*, the cost of exporting relative to the cost of innovation decreases, therefore the effect trade has on innovation will be differentiated according to the level of transportation costs. On the one hand, there is an effect through the intensive margin of innovation. The innovation intensity increases with the participation in foreign markets and thus, the effect will be larger in the *low cost trade equilibrium* where the economy is more open. On the other hand, there is an effect through the extensive margin of innovation. In Crespo (2011), it is shown that the impact on average productivity through the extensive margin will be negative in the *low cost innovation equilibrium*, undetermined in the *intermediate equilibrium* and can be positive in the *low cost trade equilibrium*. In the empirical analysis we will decompose the change in productivity due to trade costs into these components and quantify their relevance.

Equilibrium	Conditions
Low Cost Innovation Equilibrium	$\tau^{\sigma-1} f_X \geq \frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]}{n\tau^{1-\sigma}} f_I + \left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}} (\alpha+1)$ & $\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}} (\alpha+1) \geq f_D$
Intermediate Equilibrium	$\frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]}{n\tau^{1-\sigma}} f_I + \left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}} (\alpha+1) \geq \tau^{\sigma-1} f_X$ & $\tau^{\sigma-1} f_X \geq \frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{(1+n\tau^{1-\sigma})} \geq f_D$
Low Cost Trade Equilibrium	$\frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{(1+n\tau^{1-\sigma})} \geq \tau^{\sigma-1} f_X \geq f_D$

Table 1: Equilibria in the Open Economy

4 Calibration

I calibrate the model to match a number of salient features of innovation, the firm size distribution and international trade in five European countries, using firm-level data survey by the EFIGE project. The sample includes around 3,000 firms for France, Germany, Italy and Spain, and more than 2,200 firms for the United Kingdom. The survey, conducted during the year 2009, contains both qualitative and quantitative information data on firms' characteristics and activities in 2008. The distribution by firm size for the sample and the reference population are shown for each country in Table 2.[6]

Country	Between 10 and 49 S	P	Between 50 and 249 S	P	More than 250 S	P	Total S	P
France	2,151	32,019	608	7,365	214	1,986	2973	41,370
Germany	1836	52,489	793	16,988	306	3,970	2,935	79,144
Italy	2,447	77,092	429	10,062	145	1,408	3,021	88,562
Spain	2,280	38,116	406	6,241	146	1,010	2,832	45,367
U.K.	1,515	27,187	529	7,794	112	1,758	2,156	36,739

Table 2: Distribution by size, sample (S)/reference population(P)

Parameters common to all countries are taken directly from the empirical literature, while parameters specific to each country are calibrated such that particular firm-level moments in the model match those moments in the data. Parameters common to all countries are the elasticity of substitution, the elasticity of innovation, the probability of firm exit and the sunk cost of entry. The elasticity of substitution is set to be consistent with empirical estimates provided by Broda and Weinstein (2006). The medians reported vary from 2.2 to 4.8 depending on the level of aggregation, thus I set $\sigma = 3.2$ which lies within the estimated range of values. The innovation parameter α is taken to be 0.9. This is consistent with the estimate of Rubini (2011), who sets the elasticity of productivity to resources devoted to innovation to match a 5% gain in labor productivity in Canada following the tariff reduction in the U.S.-Canada Free Trade Agreement between 1980 and 1996. The probability of exit and the sunk cost of entry determine the entry and exit of firms. Following Bernard et al. (2007) I set them to $\delta = 0.025$ and $f_E = 2$.

Parameters specific to each country are innovation fixed costs (f_I), export fixed costs (f_X), variable trade costs (τ), domestic fixed costs (f_D), the productivity distribution, and the number of trading partners. The first four are calibrated jointly to match the number of workers in innovation, the percentage of exporters innovators in the economy, the ratio of exports to revenue and the percentage of executives (including entrepreneurs and middle

[6]The sample design over-represents large firms, therefore sampling weights have been constructed in terms of size-sector cells to make the sample representative of the underlying population. The calibration is based on the weighted sample.

management) in the labor force. To match the productivity distribution, I target the slope of the firm size distribution in terms of employees, and similarly to Helpman et al. (2004) and Chaney (2008), I assume the productivity is distributed according to a Pareto with a probability density function

$$g(\varphi) = \frac{\theta}{\varphi^{\theta+1}}$$

where $\varphi \in [1, \infty)$ and θ is the curvature parameter. In accordance to the model considered, I estimate by maximum likelihood the curvature parameter associated to the distribution of firms, $\tilde{\theta} = \theta/(\sigma - 1)\left(\frac{\alpha+1}{\alpha}\right)$. Given that the model assumes symmetric country sizes, the number of a country's trading partners is determined by the country's size relative to the size of the other countries. For example, if the number of employees in Spain is one-eighth the number of employees in the rest of the countries, we assume Spain has 8 trading partners. The targets are reported in Table 3.

Country	Slope	Employees	Executives	Export Volume	Exporters Innovators	R&D Workers
France	1.06	2,903,820	17.4%	27.30%	22.82%	6.81%
Germany	1.10	5,565,414	9.3%	19.48%	27.59%	6.16%
Italy	1.43	3,555,052	7.6%	32.81%	27.73%	5.81%
Spain	1.27	2,010,424	9.5%	21.50%	19.89%	4.85%
U.K.	1.01	3,729,340	14.5%	25.84%	24.31%	7.38%

Table 3: Calibration Targets

Several facts stand out in Table 3 that will help us interpret the differences in the calibrated parameters and our findings. There are important differences in export shares across countries. While exports make up 33% of revenues for Italian firms, that figure drops to 21.5% in Spain and 19.5% in Germany. Similarly, while 28% of Italian and German firms export and innovate, that share drops to 20% in Spain. The differences in R&D workers are not as substantial across countries: U.K. is the country that employs most workers in R&D (7.4%) while Spain is the country that employs least (4.85%). As for the slope of the distribution of exporting firms, a higher number indicates a steeper slope, and therefore a smaller proportion of larger firms exporting. Consistent with this, in Italy and Spain the typical exporter is relatively smaller, whereas in France and the U.K. there are many large exporting firms. The percentage of executives and middle management also differs quite a bit across countries. France and UK appear to have a more horizontal structure given that the percentage of executives (included entrepreneurs and middle management) is 17.4% and 14.5% respectively, whereas for Italian firms it drops to 7.6%, indicating a more vertical structure. The calibrated parameters for each country are in Table 4.

Country	θ	n	f_D	τ	f_X	f_I	$\Omega = f_X\tau^{\sigma-1}$	$f_X^E = nf_X$
France	4.9	6	0.95	1.88	0.44	5.75	1.76	2.64
Germany	5.1	2	2	1.14	8.4	10.6	11.2	16.8
Italy	6.6	4	1.5	1.19	5.5	6	8.1	22
Spain	5.9	8	2	1.93	4.3	2.55	18.3	34.4
U.K.	4.7	4	1.25	1.68	0.6	8.5	1.88	2.4

Table 4: Calibrated Parameters

Several of these results require some further explanation. First, Germany's fixed trade costs are relatively high with respect to other countries such as Spain, in spite of being a more open economy. This is easily explained by the fact that f_X represents the fixed trade cost paid by export destination. Because Germany's domestic market is much larger than Spain's, our assumption on symmetric countries implies that Germany has 2 trading partners, compared to 8 in the case of Spain. Therefore, as shown in Table 4, the effective fixed trade costs of a German exporter is 16.8, while the effective fixed trade costs of a Spanish exporter is 34.4 labor units. Second, France has a relatively high variable trade cost, similar to Spain, but this is partly offset by the relatively low fixed export cost. Finally, in spite of Spanish innovation fixed costs being the lowest, this does not imply higher innovation. In Spain, exporting is a very expensive activity in comparison to innovation, which explains why some domestic firms innovate without exporting. However, those firms innovate less intensively than the exporter innovators, so that the overall intensity of innovation in Spain is lower than in other countries.

The calibration predicts in which of the three equilibria described in the theory is each of the countries considered. The prediction is in Table 5, each equilibrium is determined by the openness of the economy and the level of innovation. The openness depends on both the fixed and the variable trade cost. The parameter Ω in Table 4 captures their joint effect, so that a country with a lower Ω is more open. In agreement with the theory, France and United Kingdom, the most open countries with relatively high innovation, are in the *low cost trade equilibrium*. Germany and Italy, which are less open and have average innovation are in the *intermediate equilibrium*. Spain, the most closed and least innovative country of the five, is in the *low cost innovation equilibrium*.

Country	Predicted Equilibrium
France	Low Cost Trade Equilibrium
Germany	Intermediate Equilibrium
Italy	Intermediate Equilibrium
Spain	Low Cost Innovation Equilibrium
U.K.	Low Cost Trade Equilibrium

Table 5: Predicted Equilibrium

5 Numerical Results

In the numerical analysis I consider the effect on aggregate productivity and welfare of the following experiments: a decrease in variable trade costs, a decrease in fixed trade costs, and a decrease in fixed innovation costs.

The theory previously described predicts that a decrease in variable trade cost can have a substantial impact on individual firms' decisions, and thus on aggregate productivity. In addition to the direct effects on productivity, it identifies three more channels through which indirect productivity gains can happen: the selection effect, the extensive margin of innovation and the intensive margin of innovation. The first quantitative exercise focuses on the decomposition of the change in aggregate productivity into these components and quantifying their relevance. The second quantitative exercise focuses on the effect of lowering the fixed costs of trade and innovation on productivity. Much of the literature has limited its attention to the decrease in variable trade costs. However, in a model with both trade and innovation, liberalizing trade by lowering fixed costs or by reducing variable costs may have very different results.

The section is structured as follows. First, I define the aggregate productivity measure used in the quantitative exercises, as well as its relation to welfare, following the definition of Atkeson and Burstein (2010). Second, I decompose changes in aggregate productivity following a drop in variable trade costs into its different components. Finally, I analyze the effectiveness of a trade liberalization policy versus the effectiveness of an innovation policy on aggregate productivity.

5.1 Aggregate Productivity

Assume the economy is in steady-state. To solve for aggregate quantities we define indices of aggregate productivity across firms implied by firms decisions. The first of these, Ψ_D, is an index of productivity aggregated across all operating, non-exporting domestic firms, excluding their innovation activities:

$$\Psi_D = \int_{\varphi_D}^{\varphi_X} \varphi^{\sigma-1} dG(\varphi)$$

The second, Ψ_X, is an index of productivity aggregated across all exporting domestic firms, excluding their innovation activities:

$$\Psi_X = \int_{\varphi_X}^{\infty} \varphi^{\sigma-1} dG(\varphi)$$

The third, Ψ_I, is an index of the productivity coming exclusively from the innovation activities. Since in some equilibria there are only exporter innovators, while in others there are exporter and domestic innovators, Ψ_I is defined slightly differently for each of the equilibria:

$$\Psi_I^{LCIE} = \int_{\varphi_{DI}}^{\varphi_{XI}} \varphi^{(\sigma-1)\left(\frac{\alpha}{\alpha+1}\right)} dG(\varphi) + \left[1 + \tau^{(1-\sigma)}\right]^{\frac{\alpha+1}{\alpha}} \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha}{\alpha+1}\right)} dG(\varphi)$$
$$\Psi_I^{IE} = \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha}{\alpha+1}\right)} dG(\varphi)$$
$$\Psi_I^{LCTE} = \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha}{\alpha+1}\right)} dG(\varphi)$$

where the superscripts $LCIE$, IE and $LCTE$ refer to, respectively, the *low cost innovation equilibrium*, *the intermediate equilibrium*, and *the low cost trade equilibrium*.

The output per production worker measures aggregate productivity, Ψ, whereas the output per worker measures welfare, W. Both measures can be expressed as a function of the productivity indices previously described:

$$\Psi = \frac{Q}{L_p} = \left[M\left(\Psi_D + \left(1+\tau^{1-\sigma}\right)\Psi_X + F(\tau)I\Psi_I\right)\right]^{\frac{1}{\sigma-1}} \tag{15}$$

$$W = \frac{Q}{L} = \left(\frac{\sigma-1}{\sigma}\right)\left[M\left(\Psi_D + \left(1+\tau^{1-\sigma}\right)\Psi_X + F(\tau)I\Psi_I\right)\right]^{\frac{1}{\sigma-1}} \tag{16}$$

where I is the minimum level of innovation of an innovating firm in each equilibrium, and $F(\tau)$ is a function of the variable trade costs different in each equilibrium. Appendix D provides a formal derivation of the aggregate productivity in the different equilibria. I focus on this measure of productivity because it is the measure of productivity that is relevant for welfare in our model and is similar to the ideal measure of productivity defined by Atkeson and Burstein (2010), hence making our results comparable.[7]

5.2 Decomposing the Productivity Effect of a Reduction in Variable Trade Costs

In this section, I analytically and quantitatively study the impact of a change in marginal trade costs on the measure of aggregate productivity. Following Atkeson and Burstein (2010), I do a first order approximation of the effect of a reduction in marginal international trade costs τ, decomposing its impact on productivity into a direct effect and an indirect effect. The direct effect takes all firms' decisions as given, and simply measures the productivity gains from trade being less wasteful, whereas the indirect effect arises from changes in firms' entry, export and innovation decisions, which are themselves responding to the change in trade costs. The following proposition shows the decomposition.

Proposition 4. *The total change in productivity from a change in trade costs and be decomposed into a direct effect and an indirect effect. Moreover, the indirect effect can be*

[7] This measure of aggregate productivity does not necessarily correspond to aggregate productivity as measured in the data. If all differentiated products are intermediate goods used in production of final goods, changes in the price level for final expenditures can be directly measure using final goods and $\Delta log\Psi$ is the variation of measured productivity. If all different products are consumed directly as final goods, then the problem of measuring changes in the price level for final expenditures is more complicated. See Atkeson and Burstein (2010) and Bajona et al. (2008) for a discussion of related issues.

decomposed into an entry effect, a reallocation effect, and an innovation effect.

$$\left.\Delta log\Psi = \underbrace{-s_X\Delta log(\tau)}_{Exports} - \underbrace{\left(\frac{\Delta F(\tau)}{\tau}\right) s_{Inn} I \Delta log(\tau)}_{Exporters'\ Innovation}\right\} \begin{matrix}Direct\\ Effect\end{matrix}$$

$$\left.\begin{aligned}+\frac{1}{\sigma-1}\Bigg[&\underbrace{\Delta log(M)}_{Entry\ Effect} + \underbrace{s_D\Delta log\left(\Psi_D\right)}_{Domestic\ Market} + \underbrace{\left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right) s_X\Delta log\left(\Psi_X\right)}_{Export\ Market}\\ &+ \underbrace{\underbrace{s_{Inn}I\Delta log(I)}_{Extensive\ Margin} + \underbrace{s_{Inn}I\Delta log(\Psi_I)}_{Intensive\ Margin}\Bigg]}_{Innovation}\end{aligned}\right\} \begin{matrix}Indirect\\ Effect\end{matrix}$$

Proof. Since in each equilibria the decisions on innovation are different, I use a general syntax to point out the different components of the decomposition. The exact equations along with the full proof are in Appendix D. In what follows, I sketch briefly the mathematics behind the decomposition [8].

Recall that for every $x \in \mathcal{R}$

$$\frac{\Delta x}{x} = \Delta log(x)$$

Take logs of Ψ

$$\Psi = \frac{1}{\sigma-1}\left[log(M) + log\left(\Psi_D + \left(1+\tau^{1-\sigma}\right)\Psi_X + F(\tau)I\Psi_I\right)\right]$$

And derivatives

$$\Delta log\Psi = \frac{1}{\sigma-1}\left[\Delta log(M) + \Delta log\hat{\Psi}\right]$$

$$\begin{aligned}\Delta log\hat{\Psi} = \frac{1}{\hat{\Psi}}\big[&\Delta\Psi_D + \Delta\left(1+\tau^{1-\sigma}\right)\Psi_X + \left(1+\tau^{1-\sigma}\right)\Delta\Psi_X\\ &+\Delta F(\tau)I\Psi_I + F(\tau)\Delta I\Psi_I + F(\tau)I\Delta\Psi_I\big]\end{aligned}$$

Define the share of domestic production excluding innovation in the value of production

[8] This derivation works well only for infinitesimal changes

$s_D = \frac{\Psi_D}{\hat{\Psi}}$, the share of export production excluding innovation in the value of production $s_X = \frac{n\tau^{1-\sigma}\Psi_X}{\hat{\Psi}}$ and the share of exporters innovation activities in the value of production $s_{XI}^{LCI} = \frac{\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\Psi_{XI}}{\hat{\Psi}}$ and $s_{XI}^{IE,LCT} = \frac{\left(1+n\tau^{1-\sigma}\right)\Psi_{XI}}{\hat{\Psi}}$.

□

The purpose of the decomposition is to test the prediction of the theoretical model and to quantify the importance of the different effects. I now discuss each effect, and its expected theoretical sign. The direct effect takes all firms' decisions as given and has two positive components: the first captures the productivity gain of exporters which lose less output from exporting, and the second captures the additional return from innovation by exporters that now face lower trade costs. The indirect effect has five components: the first three correspond to the selection effect described in Melitz (2003), whereas the last two correspond to the change in innovation. As for the selection effect, the first component corresponds to a drop in trade costs inducing the exit of less productive firms, implying the entry effect should be negative. The second and third components have to do with the reallocation of market shares between the remaining domestic and exporting firms. Less productive firms lose market share to more productive exporting firms, hence the domestic indirect effect should be negative and the exporters indirect effect positive. As for the innovation effect, it can be decomposed into the intensive and extensive margin of innovation. The innovation intensity increases with the participation in foreign markets and thus, the effect through the intensive margin of innovation of the exporters innovators should be positive. For the extensive margin, the theory predicts that the effect can be positive or negative. In the *low cost trade equilibrium* and the *intermediate equilibrium*, all innovators are exporting. In that case a decrease in iceberg trade costs increases the incentives to be an exporter (and to be an exporter innovator), so that the effect through the extensive margin of innovation should be positive. In the *low cost innovation equilibrium*, innovation happens by both exporting and domestic firms. Hence, while a decrease in iceberg trade costs increases the incentives of exporters to innovate, for the domestic firms innovation becomes harder, as real wages are pushed up. This implies that the productivity cutoff of domestic innovators moves to the right, so that the effect through the extensive margin of innovation will be negative.

	France	Germany	Italy	Spain	United Kingdom
Total Effect	0.643	0.642	0.806	0.650	0.597
Direct Effect	0.590	0.593	0.714	1.294	0.560
Exporter	0.021	0.017	0.055	0.031	0.005
Exporters' Innovation	0.569	0.576	0.659	1.263	0.555
Indirect Effect	0.053	0.049	0.092	−0.644	0.038
Entry	−1.182	−1.265	−2.033	−1.555	−1.115
Domestic Market	−0.010	−0.003	−0.014	−0.006	−0.003
Export Market	0.062	0.038	0.167	0.087	0.015
Innovation	1.183	1.279	1.973	0.830	1.141
Extensive Margin	0.108	0.099	0.089	−0.568	0.065
Intensive Margin	1.075	1.181	1.884	1.399	1.076
Equilibrium	LCT	IE	IE	LCI	LCT

Table 6: Elasticities Lowering Iceberg Trade Costs 1%

Table 6 shows the elasticity of each component with respect to a decrease in variable trade costs in the five countries. All the elasticities have the predicted signs. A decrease in iceberg trade costs induces in all countries an increase in total productivity. The direct effect on exporting through innovation is stronger the more closed the economy is, since they react more strongly to variations in trade costs. There is a negative effect through the entry of firms, and through the loss of market share by domestic firms, while there is a positive effect coming from the gain in market share by exporting firms and the intensive margin of innovation. Finally, as predicted, the extensive margin of innovation has a positive effect in the economies that are in the *low cost trade equilibrium* or *intermediate equilibrium*, while it is negative in the *low cost innovation equilibrium* economies.

Atkeson and Burstein (2010) predict that although a drop in iceberg trade costs changes individual firms' decisions, the total indirect effect is essentially zero. In contrast, my simulations show that this is not always the case. If the effect through the extensive margin is small, as in the case of the United Kingdom, then the indirect effect on total productivity is close to 0, since the response through the intensive margin of innovation offsets the impact of changes in firms' exit. However, if the effect through the extensive margin is large, as happens in Spain, this is no longer the case, and the indirect effect substantially differs from zero.

The difference between Atkeson and Burstein (2010) and my paper is that I have an extensive margin of innovation. Taking into account the extensive margin is particularly important in the *low cost innovation equilibrium*, where the number of total innovators in the economy decreases after a reduction of trade costs, and therefore the impact on aggregate productivity is negative. However, in all the equilibria where the impact is positive, since the number of innovators in the economy increases, the effect through the extensive margin of innovation is quite small. Consistent with this, I observe that a 1% drop in trade costs leads to a reduction of 1.84% in innovating firms in Spain (the only country in the low cost innovation equilibrium), whereas in Germany the number of innovating firms increases only by 0.41%, hence I expect a larger effect through the extensive margin of innovation in Spain than in Germany.

5.3 Lowering Fixed Costs of Trade and Innovation

The model is particularly suitable to study the effectiveness of trade and innovation policies. In this section I compare the response of aggregate productivity to a decrease in fixed trade costs versus the response to a decrease in fixed innovation costs. While much of the trade literature focuses on decreases in variable trade costs, evaluating the effect of lowering fixed costs is also important. This is especially true in model where firms take both export and innovation decisions.[9]

First, I will describe the effects of a drop in fixed trade costs and a drop in fixed innovation costs on the decisions of the firms in the economy. Second, I will quantitatively assess the elasticity of total productivity, and therefore welfare, to fixed costs. Third, I will analyze the impact on aggregate productivity of a change in the economies' equilibrium as a consequence of a large drop in fixed costs.

5.3.1 Effects on firms' decisions of a drop in fixed costs

A reduction in fixed trade costs increases the incentives to enter the export market. In the *low cost innovation equilibrium* and the *intermediate equilibrium* this implies that there is an increase in the firms that export and innovate. In the *low cost trade equilibrium* it implies that more firms export but that less firms export and innovate. In this equilibrium, the firms choosing whether to innovate or not are already exporting (and therefore are paying the fixed export costs), so they only care about innovation costs and variable trade costs. For them a drop in fixed trade costs lowers the incentives to innovate, since it induces more entry into the industry, reducing the price index and lowering the profits coming from innovation. In the next proposition I prove this latter result.

Proposition 5. *In the low cost trade equilibrium, if fixed trade costs fall*

1. *The domestic cutoff increases* $\partial\varphi_D/\partial f_X < 0$

2. *The productivity cutoff for exporting decreases* $\partial\varphi_X/\partial f_X > 0$

3. *The productivity cutoff for exporting and innovation increases* $\partial\varphi_{XI}/\partial f_X < 0$

Proof. Assume that $G(\varphi) = 1 - \left(\frac{1}{\varphi}\right)^{\theta}$. Differentiating (Equation B.2) with respect to f_X and using $\partial\varphi_X/\partial f_X = (\varphi_X/\varphi_D)\,\partial\varphi_D/\partial f_X + [1/(\sigma-1)]\,\varphi_X/f_X$ and $\partial\varphi_{XI}/\partial f_X = (\varphi_{XI}/\varphi_D)\,\partial\varphi_D/\partial f_X$ from Equation 8, Equation 9 and Equation 10 yields:

[9] In a pure trade model, without innovation, lowering variable or fixed costs tend to have qualitatively similar results on welfare. See (Melitz, 2003) for a more comprehensive explanation.

$$
\begin{aligned}
\frac{\partial \varphi_D^{LCT}}{\partial f_X} &= \frac{n\frac{1}{\varphi_X^{\theta}}}{-nf_X\left(\frac{\sigma-1}{\theta-(\sigma-1)}\right)\frac{\theta}{\varphi_X^{\theta}}\frac{1}{\varphi_D}-f_I\left(\frac{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}{\theta-(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\right)\frac{\theta}{\varphi_{XI}^{\theta}}\frac{1}{\varphi_D}}<0\\
\frac{\partial \varphi_X^{LCT}}{\partial f_X} &= \frac{1}{\theta f_X}-\frac{f_I\left(\frac{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}{\theta-(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\right)\frac{\theta}{\varphi_{XI}^{\theta+1}}\frac{\varphi_{XI}}{\varphi_X}}{nf_X\left(\frac{\sigma-1}{\theta-(\sigma-1)}\right)\frac{\theta}{\varphi_X^{\theta+1}}}\left(\frac{\varphi_X}{\varphi_D}\right)\frac{\partial \varphi_D}{\partial f_X}>0\\
\frac{\partial \varphi_{XI}^{LCT}}{\partial f_X} &= \left(\frac{\varphi_{XI}}{\varphi_D}\right)\frac{\partial \varphi_D}{\partial f_X}<0
\end{aligned}
$$

□

Similarly, a reduction in fixed innovation costs increases the incentives to start innovating. In the *low cost trade* equilibrium and the *intermediate* equilibrium this implies that there is an increase in the firms that export and innovate (because all innovators are exporting). In the *low cost innovation* equilibrium, it implies that more firms innovate but that less firms export and innovate. A drop in fixed innovation costs lowers the incentives to export, since it induces more entry into the industry, reducing the price index and the profits coming from exporting.

5.3.2 Elasticity of total productivity to fixed costs

Table 7 reports the elasticity of aggregate productivity with respect to a reduction in the fixed costs of trade and innovation, and compares them to the elasticity of aggregate productivity with respect to a reduction in the marginal trade cost. The aggregate productivity of the economy responds much more strongly to a change in marginal trade costs than to a change in fixed trade costs or fixed innovation costs. While the elasticities with respect to the fixed costs are both small, there are significant differences between them.

On the one hand, the elasticity of aggregate productivity with respect to the fixed innovation costs is very similar across countries and always positive. For countries in the *low cost trade* or the *intermediate* equilibrium, lower fixed innovation costs imply more firms exporting and innovating. However, in the *low cost innovation* equilibrium, which characterizes Spain, there are two opposing effects. While the cost of innovating has dropped, there is the negative effect coming from a reduction in the incentives to export, so that the number of exporters innovators falls. As can be seen from Table 7, the direct positive effect more than offsets the negative effect, so that the overall productivity (and welfare) increases in Spain.

On the other hand, the elasticity of aggregate productivity with respect to fixed trade costs is in absolute terms greater than the elasticity with respect to fixed innovation costs, therefore a decrease in fixed trade costs appears to be more effective than a decrease in fixed innovation costs. However, the response of aggregate productivity to a drop in fixed export costs is negative in two countries, France and United Kingdom. Both economies are in the *low cost trade* equilibrium, and Proposition 5 shows that a reduction in fixed trade costs increases the incentives to enter the export market, but lowers the incentives to innovate. The intuition is that the increased presence of foreign firms pushes up real wages, which reduces the number of innovators and the intensity of the remaining innovators. Since the investment in innovation decreases, so do the total revenues (and profits) of these firms. Therefore, there is a reallocation of market shares from the most productive firms in the economy towards slightly less productive firms (the new exporters), which lowers the total productivity of the economy and therefore welfare.

	France	Germany	Italy	Spain	U.K.
$\epsilon_{\Psi,\tau}$	0.643	0.642	0.806	0.65	0.597
ϵ_{Ψ,f_X}	-0.0156	0.0124	0.0578	0.0374	-0.0197
ϵ_{Ψ,f_I}	0.0129	0.0078	0.0155	0.0174	0.0030

Table 7: Effects of a Small Reduction in τ, f_X and f_I.

5.3.3 Effect on productivity from large changes in fixed costs

Figure 5 and Figure 6 show the response of total productivity to larger changes in fixed trade costs and fixed innovation costs. On the horizontal axes are the fixed costs (in reverse order, from high to low) and on the vertical axes is the variation in total productivity with respect to the initial total productivity. An upward-sloping schedule for a given country implies that total productivity (and therefore also welfare) increases when fixed costs drop. For each country the starting point is their initial fixed costs, and I only consider decreases.

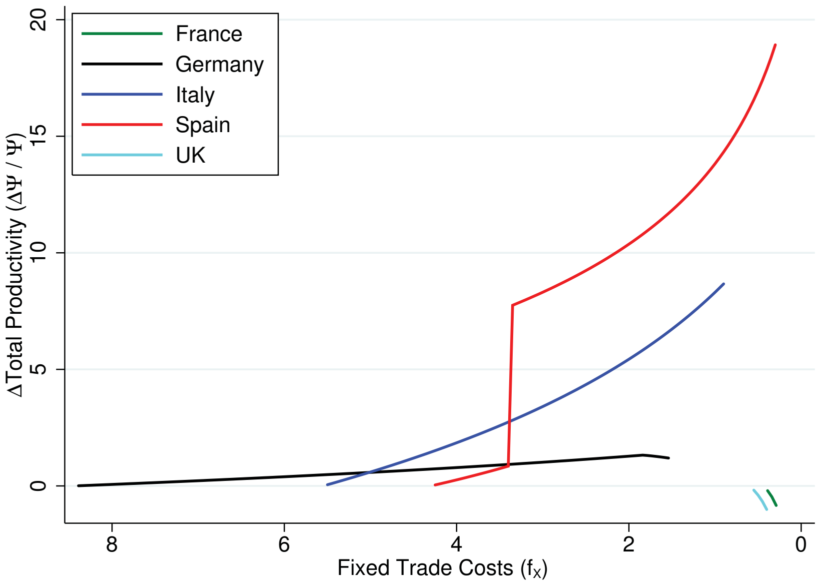

Figure 5: Change in Total Productivity and Fixed Trade Costs

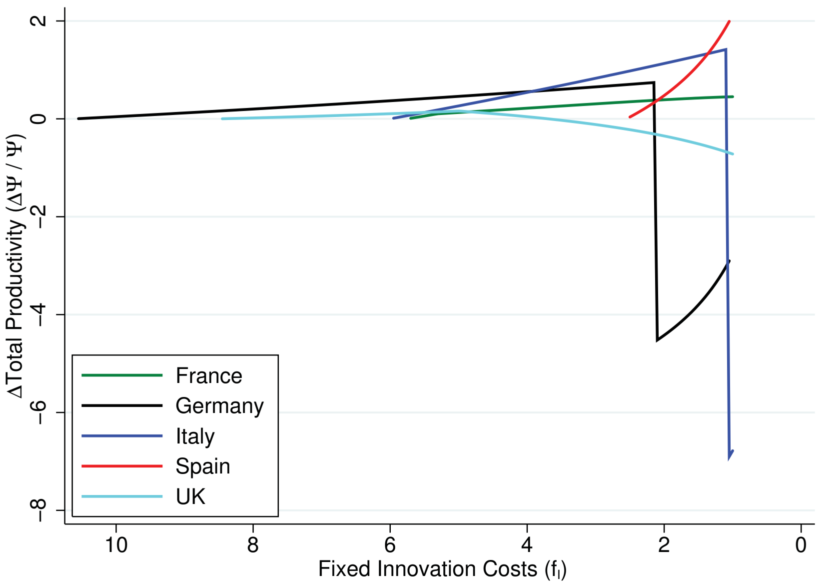

Figure 6: Change in Total Productivity and Fixed Innovation Costs

Several facts stand out in these two figures. First, the response of productivity to changes in fixed trade costs is stronger than the response to changes in fixed innovation costs. Second, if the economy is in the *low cost trade equilibrium*, the total productivity decreases as fixed trade costs decrease. This is the case of France and the UK. Third, if fixed innovation costs decrease, total productivity increases the most if the economy is the *low cost innovation equilibrium*. This the case of Spain. These three facts are similar to the ones found when computing the elasticities in Table 7.

However, the figures also reveal that the largest changes in productivity happen when countries move from one equilibrium to another as a consequence of the drop in fixed costs. This is especially relevant if the movement from one equilibrium to another has a big impact on the number of firms in the economy. These changes in productivity can be positive or negative, large or small, therefore studying what drives them is important to be able to asses the effectiveness of innovation policies and trade policies.

If the fixed trade cost drops sufficiently, Spain goes from the *low cost innovation equilibrium* to the *intermediate equilibrium*. In Figure 5 this change in equilibrium shows up as a large upward spike. In this transition 8% of the firms in the economy exit. This negative effect is more than compensated by an increase of 29% in the productivity of the economy when ignoring changes on the entry of firms. The large productivity increase is due to domestic innovators becoming exporting innovators thanks to the increased ease of entering the export market.

Similarly, if the fixed cost of innovation drops sufficiently, Italy and Germany also change equilibrium, this time in the other direction, from the *intermediate equilibrium* to the *low cost innovation equilibrium*. Once again, this shows up as a large spike in Figure 6. Since trade becomes relatively more expensive, after the transition there are less exporter innovators and more firms enter in the domestic market. The loss through the exporter innovators dominates the entry of more firms in the economy, hence the spike down in both economies during the change. Finally, notice that once in the the *low cost innovation equilibrium*, the total productivity starts increasing again.

But there are other shifts in equilibria. For example, if the fixed trade cost drops sufficiently, Germany goes from the *intermediate equilibrium* to the *low cost trade equilibrium*. And if the fixed cost of innovation drops sufficiently, France and United Kingdom go from the *low cost trade equilibrium* to the *intermediate equilibrium*. In all these cases, the change between equilibria is smooth and only the slopes change. In Figure 5, when Germany transitions to the *low cost trade equilibrium*, the trend becomes negative, although there are still gains in productivity with respect to the initial productivity since it is now in a more open economy. The negative effect is consistent with Proposition 5, where a decrease in fixed trade costs induces losses both through the extensive and the intensive margins of innovation. Note that France and the United Kingdom, which are already in the *low cost trade equilibrium*, display a similar behavior, whereby a drop in fixed trade costs lowers total productivity. However, since both of them are already in a very export oriented economy, there are no gains with respect to the initial productivity, and the decrease translates in a drop in productivity.

If we turn to the opposite case, going from the *low cost trade equilibrium* to the *intermediate equilibrium*, as France and United Kingdom do in Figure 6, we see that both countries react differently. While there is an increase of total productivity in France with respect to the initial situation, in the United Kingdom the trend is negative and if fixed innovation costs are low enough, the total productivity decreases with respect to the initial situation. The decrease in fixed innovation costs induces firms to become exporters innovators, increasing the market shares of these firms while the most inefficient exit the economy. While in France the positive effect through the reallocation of market shares towards the more efficient firms dominates the negative effect through the exit of firms, in the United Kingdom it is the negative effect through the exit of firms which dominates.

Summarizing, Figure 5 and Figure 6 reveal that a drop in fixed trade costs is more effective in raising productivity (and welfare) than a drop in fixed innovation costs. Depending on the country, it can induce productivity gains from 1% to 20% in total, and only if the economy is already very open might a further drop in fixed trade costs be damaging to the economy. In contrast, a fixed innovation cost drop has little effect on the productivity, the maximum increase being around 2%, and if it induces economies to be less export oriented, then the productivity might decrease by upto 7%.

6 Conclusions

This paper has proposed a trade model with heterogeneous firms that decide not just whether or how much to export but also whether or how much to innovate. By incorporating the extensive and intensive margins of trade and innovation, three equilibria may arise. In all equilibria high-productivity firms export and innovate, whereas low-productivity neither export nor innovate. What differs across equilibria is the behavior of medium-productivity firms. In an economy with trade costs that are low relative to innovation costs, medium-productivity firms export without innovating, whereas in an economy with trade costs that are high relative to innovation costs, medium-productivity firms innovate without exporting. In a third equilibrium, in between the other two, some medium-productivity firms export and innovate, whereas others do neither.

After characterizing these different equilibria, we have shown that they are empirically plausible by calibrating the model to five European countries. The numerical exercises reveal the importance of considering both the intensive and extensive margin of innovation to understand the interdependence between trade and innovation. More generally, the effect of trade liberalization on productivity and welfare depends crucially on the equilibrium the economy is in. A standard result in the literature is that the aggregate productivity effect of a drop in variable trade costs on firms' decisions to exit, export and innovate is minimal. In our setup this is also true in most equilibria, but not in the *low cost innovation equilibrium*. In that case a drop in variable trade costs has a negative impact on the extensive margin of innovation, thus lowering the overall positive effect of trade liberalization.

In addition to analyzing a drop in variable trade costs, I also assessed the impact of a drop in fixed trade costs and fixed innovation costs. Once again, although in most equilibria these policies lead to an improvement in aggregate productivity and welfare, this is not always the case. For example, in the *low cost trade equilibrium*, a drop in fixed trade costs increases the number of exporters, making innovating more expensive. This lowers both the number of innovators and the intensity of innovation, leading to a reduction in aggregate productivity and welfare.

These findings stress the importance of having a model that jointly analyzes the extensive and intensive margins of both trade and innovation. Not doing so would not just result in a less rich theoretical structure, it would also keep us from correctly assessing the impact of different policies aimed at fomenting trade and innovation.

Of course, this model has abstracted from a number of potentially relevant features that go beyond the scope of this paper. First, I have exclusively focused on a steady state environment, thus ignoring the transition dynamics. As shown by Alessandria and Choi (2011) and Burstein and Melitz (2011), not taking into account transition dynamics may significantly impact the welfare effects of trade liberalization. Second, the model does not consider uncertainty in innovation. While most of the literature on trade and innovation assumes there is no risk involved,[10] the empirical evidence suggests otherwise: there is

[10] An exception is Atkeson and Burstein (2010) who introduce uncertainty in the outcome of the investment in process innovation, although firms always get some returns (no innovation fails).

risk that an innovator will not identify important needs, that innovation teams disrupt the regular operations of a business, or that even a promising idea is not accepted by the customers whose need it was meant to address. Third, I have assumed that there is no strategic interaction between firms and therefore the innovation activities of one firm do not have any influence in the innovation activities of the other firms. The existence of externalities in process innovation could have a significant effect on the results. Fourth, the model could be used to analyze the effect of joint trade and innovation policies. The right mix of policies could lead to greater gains in aggregate productivity.

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Appendix A - Low Cost Innovation Economy

Productivity distribution and weighted averages

Let us denote by $\mu_D(\varphi)$, $\mu_{DI}(\varphi)$ and $\mu_{XI}(\varphi)$ respectively, the productivity distribution of domestic producers, active innovators and active innovators and exporters prior to innovation.

$$\mu_D(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_{DI})-G(\varphi_D)} & , \varphi_{DI} > \varphi \geq \varphi_D \\ 0 & , otherwise \end{cases}$$

$$\mu_{DI}(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_{XI})-G(\varphi_{DI})} & , \varphi_{XI} \geq \varphi \geq \varphi_{DI} \\ 0 & , otherwise \end{cases}$$

$$\mu_{XI}(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi_{XI})} & , \varphi \geq \varphi_{XI} \\ 0 & , otherwise \end{cases}$$

The distributions $\mu_D(\varphi)$, $\mu_{DI}(\varphi)$ and $\mu_{XI}(\varphi)$ are not affected by the simultaneous entry and exit since the successful entrants and failing incumbents draw their productivity level from the common distribution $\mu(\varphi)$.

Let $\widetilde{\varphi} = \left[\int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)\, d\varphi\right]^{\frac{1}{(\sigma-1)}}$ and $\widetilde{\varphi}_X = \left[\int_{\varphi_{XI}}^{\infty} \varphi^{\sigma-1}\mu_{XI}(\varphi)\, d\varphi\right]^{\frac{1}{(\sigma-1)}}$ denote the average productivity levels of, respectively, all firms and exporting firms only prior to innovation. Then the weighted productivity average that reflects the combined market share of all firms can be defined as

$$\widetilde{\varphi}_t = \left\{\frac{1}{M_t}\left[M\widetilde{\varphi}^{\sigma-1} + nM_X\left(\tau^{-1}\widetilde{\varphi}_X\right)^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}}$$

And let $\widetilde{\varphi}_{DI} = \left[\int_{\varphi_{DI}}^{\infty} \left(\varphi^{\sigma-1}\right)^{\frac{(\alpha+1)}{\alpha}} \mu_{DI}(\varphi)\, d\varphi\right]^{\frac{\alpha}{(\alpha+1)}\frac{1}{(\sigma-1)}}$ and $\widetilde{\varphi}_{XI}$ represent the average productivity the domestic innovators and exporter innovators get from innovation. Then the weighted productivity average that reflects the combined market share of innovation can be defined as

$$\widetilde{\varphi}_t^I = \left\{\frac{1}{M_t^I}\left[M_I\left(\widetilde{\varphi}_{DI}\right)^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)} + m_{XI}\left(\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right)\left(\widetilde{\varphi}_{XI}\right)^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\right]\right\}^{\left(\frac{\alpha}{\alpha+1}\right)\left(\frac{1}{\sigma-1}\right)}$$

Aggregate Variables

Denote by m_{XI}, m_{DI} and m_D respectively the mass of active innovators and exporters, active innovators but non-exporters and non-innovators and non-exporters present in the economy,

$$
\begin{aligned}
m_{XI} &= \frac{1-G\left(\varphi_{XI}\right)}{1-G\left(\varphi_{D}\right)}M \\
m_{DI} &= \frac{G\left(\varphi_{XI}\right)-G\left(\varphi_{DI}\right)}{1-G\left(\varphi_{D}\right)}M \\
m_{D} &= \frac{G\left(\varphi_{DI}\right)-G\left(\varphi_{D}\right)}{1-G\left(\varphi_{D}\right)}M
\end{aligned}
$$

with M being the mass of incumbent firms in the economy, $M_I = m_{DI} + m_{XI}$ the number of firms that perform innovation activities and $M_X = m_{XI}$ the number of firms performing exporting activities. The total number of varieties sold in the economy (by symmetry) will be $M_t = M + nM_X$, and the total number of varieties coming from innovators will be $M_t^I = M_I + nM_X$.

It can be shown that the aggregates will take the following expressions

- Aggregate Price Index

$$
P^{1-\sigma} = M_t\left[p_D\left(\widetilde{\varphi}_t\right)\right]^{1-\sigma} + M_t^I z_D\left(\varphi_{DI}\right)\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left[p_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)\right]^{1-\sigma}
$$

- Aggregate Production

$$
Q^{\rho} = M_t\left[q_D\left(\widetilde{\varphi}_t\right)\right]^{\rho} + M_t^I z_D\left(\varphi_{DI}\right)\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left[q_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)\right]^{\rho}
$$

- Aggregate Revenue

$$
R = M_t r_D\left(\widetilde{\varphi}_t\right) + M_t^I z_D\left(\varphi_{DI}\right)\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)
$$

- Aggregate Profits

$$\begin{aligned}\Pi &= M_t \frac{r_D\left(\widetilde{\varphi}_t\right)}{\sigma} - Mf_D - nM_X f_X - M_I f_I + M_I \alpha \left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}} \left(\frac{r_D\left(\widetilde{\varphi}_I\right)}{\sigma}\right)^{\frac{\alpha+1}{\alpha}} \\ &\quad + m_{XI}\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right]\alpha\left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}} \left(\frac{r_D\left(\widetilde{\varphi}_{XI}\right)}{\sigma}\right)^{\frac{\alpha+1}{\alpha}} \end{aligned} \tag{A.1}$$

Low Cost Innovation Equilibrium

Proof of Proposition 1, part II

If there are sufficiently high fixed export cost, there exist a single cutoff φ^*_{XI} that solves equation (6)

Proof. The proof is divided in three sections

First, I show that the LHS of equation (6) is positive with respect to the productivity parameter. $\pi_{XI}\left(\varphi_{XI}\right) - \pi_{DI}\left(\varphi_{XI}\right) \geq 0$

$$\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right]\alpha\left(\tfrac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}} \left[\left(\tfrac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}} + n\tau^{1-\sigma}\left(\tfrac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1} - nf_x \geq 0$$

$$C_1\left(\varphi^{\sigma-1}\right)^{\frac{\alpha+1}{\alpha}} + C_2\varphi^{\sigma-1} - nf_x \geq 0$$

$$\frac{\partial LHS}{\partial \varphi} = C_1\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)\,\varphi^{\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)-1} + C_2(\sigma-1)\,\varphi^{\sigma-2} > 0$$

Secondly, I show that $\pi_{XI}\left(\varphi_{DI}\right) - \pi_{DI}\left(\varphi_{DI}\right) < 0$, otherwise the firm would choose to export and innovate instead of being indifferent between innovating or not while staying in the domestic market.

$$\begin{aligned}&\pi_{XI}\left(\varphi_{DI}\right) - \pi_{DI}\left(\varphi_{DI}\right) < 0 \\ &\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right] f_I + n\tau^{1-\sigma}\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1) - nf_X < 0\end{aligned}$$

Thus, for f_X large enough, that is for

$$f_X > \left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right]\frac{f_I}{n} + \tau^{1-\sigma}\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)$$

it holds that $\pi_{XI}\left(\varphi_{DI}\right) - \pi_{DI}\left(\varphi_{DI}\right) < 0$

Finally, I show that the difference between the profits of the exporting and non-exporting strategies while innovation goes to infinite as the productivity of the firm is larger.

If $\varphi \to \infty$,then $\pi_{XI}(z_D(\varphi)) - \pi_{DI}(z_D(\varphi)) \to \infty$, since by definition $\pi_{XI}(z_X(\varphi)) > \pi_{XI}(z_D(\varphi))$ then it must be that $\pi_{XI}(z_X(\varphi)) - \pi_{DI}(z_D(\varphi)) \to \infty$ as $\varphi \to \infty$

$$\begin{aligned}\pi_{XI}(z_D(\varphi)) - \pi_{DI}(z_D(\varphi)) &= n\tau^{1-\sigma}[1+z]\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1} - nf_X \\ &= n\tau^{1-\sigma}\left(\frac{1}{\alpha+1}\right)^{\frac{1}{\alpha}}\left[\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}} \\ &\quad + n\tau^{1-\sigma}\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1} - nf_X\end{aligned}$$

$$\begin{aligned}\lim_{\varphi\to\infty}\left[\pi_{XI}(z_D(\varphi)) - \pi_{DI}(z_D(\varphi))\right] &= \lim_{\varphi\to\infty}\left[C_4\left[\varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}} + C_5\varphi^{\sigma-1} - C_6\right] \\ &= \lim_{\varphi\to\infty}\left[C_4\left[\varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}\right] + \lim_{\varphi\to\infty}\left[C_5\varphi^{\sigma-1}\right] - \lim_{\varphi\to\infty}(C_6) \to \infty\end{aligned}$$

□

Proof of Proposition 1, part I

Equations (4) to (6) along with the Free Entry condition (7) completely determine the equilibrium and the productivity cutoffs can be uniquely determined and allow me to rearrange the FE conveniently for the characterizing of the equilibrium as a function of φ_D^*

$$\delta f_E = [1 - G(\varphi_D^*)]\,\overline{\pi}$$

$$\begin{aligned}\delta f_E &= f_D j_1(\varphi_D^*) + n\tau^{1-\sigma} f_D j_2(\varphi_X^*(\varphi_D^*)) - [1 - G(\varphi_{XI}^*)]\, n f_X \\ &\quad - [1 - G(\varphi_{DI}^*)]\, f_I + \alpha\left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}} f_D^{\frac{\alpha+1}{\alpha}} j_3(\varphi_D^*) \\ &\quad + \alpha\left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}} f_D^{\frac{\alpha+1}{\alpha}}\left[\left(1 + n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right] j_4(\varphi_D^*)\end{aligned} \tag{A.2}$$

where $j_1(\varphi_D^*) = \left[\left(\widetilde{\varphi}(\varphi_D^*)/\varphi_D^*\right)^{\sigma-1} - 1\right]$, $j_2(\varphi_D^*) = \left(\widetilde{\varphi}_x(\varphi_D^*)/\varphi_D^*\right)^{\sigma-1}\left[1 - G(\varphi_{XI}^*)\right]$

$j_3(\varphi_D^*) = \left[\left(\widetilde{\varphi}_{DI}(\varphi_D^*)/\varphi_D^*\right)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}\left[1 - G(\varphi_{DI}^*)\right]$ and

$j_4(\varphi_D^*) = \left[\left(\widetilde{\varphi}_{XI}(\varphi_D^*)/\varphi_D^*\right)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}\left[1 - G(\varphi_{XI}^*)\right]$

Proof.

Assume the parameter restrictions $\tau^{\sigma-1} f_X \geq \frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right]}{n\tau^{1-\sigma}} f_I + \left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)$ and $\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1) \geq f_D$ hold, then the Low Cost Innovation Equilibrium exists and is unique.
I shall proof that the RHS of equation ($A.2$) is decreasing in φ_D^* on the domain (φ_D^*, ∞), so that φ_D^* is uniquely determined by the intersection of the latter curve with the flat line δf_E in the (φ_D^*, ∞) space.
Let $k_1(\varphi_D^*) = \left[\left(\widetilde{\varphi}(\varphi_D^*)/\varphi_D^*\right)^{\sigma-1} - 1\right]$, then

$$k_1'(\varphi_D^*) = \frac{g(\varphi_D^*)}{1 - G(\varphi_D^*)} k_1(\varphi_D^*) - \frac{(\sigma-1)\left[k_1(\varphi_D^*) + 1\right]}{\varphi_D^*}$$

Similarly, $k_3(\varphi_D^*) = \left[\left(\widetilde{\varphi}_{DI}(\varphi_D^*)/\varphi_D^*\right)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}$, thus

$$k_3'(\varphi_D^*) = \Lambda^{\frac{1}{\sigma-1}} \frac{g(\varphi_I^*)}{1 - G(\varphi_I^*)}\left[k_2(\varphi_D^*) - \Lambda^{\frac{\alpha+1}{\alpha}}\right] - \left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)\frac{k_2(\varphi_D^*)}{\varphi_D^*}$$

where $\frac{\partial \varphi_{DI}^*}{\partial \varphi_D^*} = \left[\frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{f_D}\right]^{\frac{1}{\sigma-1}} = \Lambda^{\frac{1}{\sigma-1}}$

Now, define $j_1(\varphi_D^*) = \left[1 - G(\varphi_D^*)\right] k_1(\varphi_D^*)$, and $j_3(\varphi_D^*) = \left[1 - G(\varphi_{DI}^*)\right] k_3(\varphi_D^*)$ which are non-negative.
Then the derivative and elasticity of $j_1(\varphi_D^*)$ and $j_3(\varphi_D^*)$ are

$$j_1'(\varphi_D^*) = -\frac{(\sigma-1)\left[k_1(\varphi_D^*) + 1\right]}{\varphi_D^*}\left[1 - G(\varphi_D^*)\right] < 0$$

$$\frac{j_1'(\varphi_D^*) \cdot \varphi_D^*}{j_1(\varphi_D^*)} = \underbrace{-(\sigma-1)\left[1 + \frac{1}{k_1(\varphi_D^*)}\right]}_{<0 \text{ and bounded away of it}} < -(\sigma-1)$$

and

$$j_3'\left(\varphi_D^*\right) = -g\left(\varphi_{DI}^*\right)\Lambda^{\frac{1}{\sigma-1}}\Lambda^{\frac{\alpha+1}{\alpha}} - \theta\left(\alpha+1\right)\left(\sigma-1\right)\frac{k_3\left(\varphi_D^*\right)}{\varphi_D^*}\left[1-G\left(\varphi_{DI}^*\right)\right] < 0$$

$$\frac{j_3'\left(\varphi_D^*\right)\cdot\varphi_D^*}{j_3\left(\varphi_D^*\right)} = \underbrace{-\frac{g\left(\varphi_{DI}^*\right)}{\left[1-G\left(\varphi_{DI}^*\right)\right]}\frac{\Lambda^{\frac{1}{\sigma-1}}\Lambda^{\frac{\alpha+1}{\alpha}}}{k_2\left(\varphi_D^*\right)}\varphi_D^* - \beta\left(\sigma-1\right)}_{<0 \text{ and bounded away of it}} < -\beta\left(\sigma-1\right)$$

Thus, $j_1\left(\varphi_D^*\right)$ and $j_3\left(\varphi_D^*\right)$ must be decreasing to zero as φ goes to infinite. Furthermore, it must be that $\lim_{\varphi_D^*\to 0} j_1\left(\varphi_D^*\right) = \infty$ since $\lim_{\varphi_D^*\to 0} k_1\left(\varphi_D^*\right) = \infty$.and $\lim_{\varphi_D^*\to 0} j_3\left(\varphi_D^*\right) = \infty$ since $\lim_{\varphi_D^*\to 0} k_3\left(\varphi_D^*\right) = \infty$
Since $j_1\left(\varphi_D^*\right)$ and $j_3\left(\varphi_D^*\right)$, it follows that $j_2\left(\varphi_D^*\right)$ and $j_4\left(\varphi_D^*\right)$ do also monotonically decrease from infinite to zero on the $(0,\infty)$ parameter space.

Therefore, the RHS of ($A.2$) is a monotonic decreasing function from infinity to zero on the space $(0,\infty)$ that cuts the FE flat line from above identifying a unique cutoff level φ_D^*. □

Appendix B - Low Cost Trade Economy

Productivity distribution and weighted averages

Let us denote by $\mu_D(\varphi)$, $\mu_X(\varphi)$ and $\mu_{XI}(\varphi)$ respectively, the productivity distribution of domestic producers, exporters and innovators exporters.

$$\mu_D(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_X)-G(\varphi_D)} & ,\varphi_X > \varphi \geq \varphi_D \\ 0 & ,otherwise \end{cases}$$

$$\mu_X(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_{XI})-G(\varphi_X)} & ,\varphi_{XI} \geq \varphi \geq \varphi_X \\ 0 & ,otherwise \end{cases}$$

$$\mu_{XI}(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi_{XI})} & ,\varphi \geq \varphi_{XI} \\ 0 & ,otherwise \end{cases}$$

The distributions $\mu_D(\varphi)$, $\mu_X(\varphi)$ and $\mu_{XI}(\varphi)$ are not affected by the simultaneous entry and exit since the successful entrants and failing incumbents draw their productivity level from the common distribution $\mu(\varphi)$.

Let $\widetilde{\varphi} = \left[\int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)\, d\varphi\right]^{\frac{1}{(\sigma-1)}}$ and $\widetilde{\varphi}_X = \left[\int_{\varphi_{XI}}^{\infty} \varphi^{\sigma-1}\mu_{XI}(\varphi)\, d\varphi\right]^{\frac{1}{(\sigma-1)}}$ denote the average productivity levels of, respectively, all firms and exporting firms only prior to innovation. Then the weighted productivity average that reflects the combined market share of all firms can be defined as

$$\widetilde{\varphi}_t = \left\{\frac{1}{M_t}\left[M\widetilde{\varphi}^{\sigma-1} + nM_X\left(\tau^{-1}\widetilde{\varphi}_X\right)^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}}$$

And let $\widetilde{\varphi}_{XI} = \left[\int_{\varphi_{XI}}^{\infty} \left(\varphi^{\sigma-1}\right)^{\frac{(\alpha+1)}{\alpha}} \mu_{XI}(\varphi)\, d\varphi\right]^{\frac{\alpha}{(\alpha+1)}\frac{1}{(\sigma-1)}}$ represent the average productivity the innovators get from innovation.

Aggregate Variables

Denote by m_{XI}, m_X and m_D respectively the mass of active innovators and exporters, only exporters and non-innovators non-exporters present in the economy,

$$m_{XI} = \frac{1-G(\varphi_{XI})}{1-G(\varphi_D)}M$$

$$m_X = \frac{G(\varphi_{XI})-G(\varphi_X)}{1-G(\varphi_D)}M$$

$$m_D = \frac{G(\varphi_X) - G(\varphi_D)}{1 - G(\varphi_D)} M$$

with M being the mass of incumbent firms in the economy, $M_I = m_{XI}$ the number of firms that perform innovation activities and $M_X = m_X + m_{XI}$ the number of firms performing exporting activities. The total number of varieties sold in the economy (by symmetry) will be $M_t = M + nM_X$.

It can be shown that the aggregates will take the following expressions

- Aggregate Price Index

$$P^{1-\sigma} = M_t \left[p_D\left(\widetilde{\varphi}_t\right)\right]^{1-\sigma} + m_{XI}\left(1 + n\tau^{1-\sigma}\right) z_X\left(\varphi_{XI}\right)\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left[p_D\left(\widetilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right)\right]^{1-\sigma}$$

- Aggregate Production

$$Q^{\rho} = M_t \left[q_D\left(\widetilde{\varphi}_t\right)\right]^{\rho} + m_{XI}\left(1 + n\tau^{1-\sigma}\right) z_X\left(\varphi_{XI}\right)\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left[q_D\left(\widetilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right)\right]^{\rho}$$

- Aggregate Revenue

$$R = M_t r_D\left(\widetilde{\varphi}_t\right) + m_{XI}\left(1 + n\tau^{1-\sigma}\right) z_X\left(\varphi_{XI}\right)\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\widetilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right)$$

- Aggregate Profits

$$\begin{aligned}\Pi &= M_t \frac{r_D\left(\widetilde{\varphi}_t\right)}{\sigma} - M f_D - n M_X f_X - m_{XI} f_I \\ &\quad + m_{XI}\left(1 + n\tau^{1-\sigma}\right)\alpha\left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}}\left(\frac{r_D\left(\widetilde{\varphi}_I\right)}{\sigma}\right)^{\frac{\alpha+1}{\alpha}}\end{aligned} \quad \text{(B.1)}$$

Low Cost Trade Economy Equilibrium

Proof of Proposition 2

Equations (8) to (10) along with the Free Entry condition (11)completely determine the equilibrium and the productivity cutoffs can be uniquely determined and I can rearrange the FE conveniently for the characterizing of the equilibrium as a function of φ_D^*

$$\delta f_E = [1 - G(\varphi_D^*)]\,\overline{\pi}$$

$$\begin{aligned} \delta f_E &= f_D l_1(\varphi_D^*) + n f_X l_2(\varphi_X^*(\varphi_D^*)) \\ &+\alpha\left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}}\left[f_D\left(1+\tau^{1-\sigma}\right)\right]^{\left(\frac{\alpha+1}{\alpha}\right)} l_3(\varphi_D^*) - [1 - G(\varphi_{XI}^*)]\, f_I \end{aligned} \tag{B.2}$$

where $j_1(\varphi_D^*) = \left[(\widetilde{\varphi}(\varphi_D^*)/\varphi_D^*)^{\sigma-1} - 1\right]$, $j_2(\varphi_X^*(\varphi_D^*)) = \left[(\widetilde{\varphi}(\varphi_X^*)/\varphi_X^*)^{\sigma-1} - 1\right][1 - G(\varphi_X^*)]$ and $j_3(\varphi_D^*) = \left[(\widetilde{\varphi}_{XI}(\varphi_D^*)/\varphi_D^*)^{\sigma-1}\right]^{\left(\frac{\alpha+1}{\alpha}\right)}[1 - G(\varphi_{XI}^*)]$

Proof.

Assume the parameter restriction $\frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{(1+n\tau^{1-\sigma})} \geq \tau^{\sigma-1} f_X \geq f_D$ holds, then the Low Cost Trade Equilibrium exists and is unique. I shall proof that the RHS of equation ($B.2$) is decreasing in φ_D^* on the domain (φ_D^*, ∞), so that φ_D^* is uniquely determined by the intersection of the latter curve with the flat line δf_E in the (φ_D^*, ∞) space.
Let $k_1(\varphi_D^*) = \left[(\widetilde{\varphi}(\varphi_D^*)/\varphi_D^*)^{\sigma-1} - 1\right]$, then

$$k_1'(\varphi_D^*) = \frac{g(\varphi_D^*)}{1 - G\left(\varphi_D^*\right)} k_1(\varphi_D^*) - \frac{(\sigma-1)[k_1(\varphi_D^*) + 1]}{\varphi_D^*}$$

Similarly, $k_3(\varphi_D^*) = \left[(\widetilde{\varphi}_{DI}(\varphi_D^*)/\varphi_D^*)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}$, thus

$$k_3'(\varphi_D^*) = \Lambda^{\frac{1}{\sigma-1}} \frac{g(\varphi_I^*)}{1 - G\left(\varphi_I^*\right)}\left[k_2(\varphi_D^*) - \Lambda^{\frac{\alpha+1}{\alpha}}\right] - \left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)\frac{k_2(\varphi_D^*)}{\varphi_D^*}$$

where $\frac{\partial \varphi_{DI}^*}{\partial \varphi_D^*} = \left[\frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{f_D}\right]^{\frac{1}{\sigma-1}} = \Lambda^{\frac{1}{\sigma-1}}$
Now, define $j_1(\varphi_D^*) = [1 - G(\varphi_D^*)]\, k_1(\varphi_D^*)$, and $j_2(\varphi_D^*) = [1 - G(\varphi_{DI}^*)]\, k_2(\varphi_D^*)$ which are non-negative.
Then the derivative and elasticity of $j_1(\varphi_D^*)$ and $j_3(\varphi_D^*)$ are

$$j_1'(\varphi_D^*) = -\frac{(\sigma-1)[k_1(\varphi_D^*) + 1]}{\varphi_D^*}[1 - G(\varphi_D^*)] < 0$$

$$\frac{j_1'\left(\varphi_D^*\right)\cdot\varphi_D^*}{j_1\left(\varphi_D^*\right)} = \underbrace{-\left(\sigma-1\right)\left[1+\frac{1}{k_1\left(\varphi_D^*\right)}\right]}_{<0 \text{ and bounded away of it}} < -\left(\sigma-1\right)$$

and

$$j_3'\left(\varphi_D^*\right) = -g\left(\varphi_{DI}^*\right)\Lambda^{\frac{1}{\sigma-1}}\Lambda^{\frac{\alpha+1}{\alpha}} - \theta\left(\alpha+1\right)\left(\sigma-1\right)\frac{k_3\left(\varphi_D^*\right)}{\varphi_D^*}\left[1-G\left(\varphi_{DI}^*\right)\right] < 0$$

$$\frac{j_3'\left(\varphi_D^*\right)\cdot\varphi_D^*}{j_3\left(\varphi_D^*\right)} = \underbrace{-\frac{g\left(\varphi_{DI}^*\right)}{\left[1-G\left(\varphi_{DI}^*\right)\right]}\frac{\Lambda^{\frac{1}{\sigma-1}}\Lambda^{\frac{\alpha+1}{\alpha}}}{k_2\left(\varphi_D^*\right)}\varphi_D^* - \beta\left(\sigma-1\right)}_{<0 \text{ and bounded away of it}} < -\beta\left(\sigma-1\right)$$

Thus, $j_1\left(\varphi_D^*\right)$ and $j_3\left(\varphi_D^*\right)$ must be decreasing to zero as φ goes to infinite. Furthermore, it must be that $\lim_{\varphi_D^*\to 0} j_1\left(\varphi_D^*\right) = \infty$ since $\lim_{\varphi_D^*\to 0} k_1\left(\varphi_D^*\right) = \infty$ and $\lim_{\varphi_D^*\to 0} j_3\left(\varphi_D^*\right) = \infty$ since $\lim_{\varphi_D^*\to 0} k_3\left(\varphi_D^*\right) = \infty$. Since $j_1\left(\varphi_D^*\right)$ and $j_3\left(\varphi_D^*\right)$ are decreasing from infinity to zero on $(0,\infty)$,from the closed economy case, it follows that $j_2\left(\varphi_X^*\left(\varphi_D^*\right)\right)$ does also monotonically decrease from infinite to zero on the $(0,\infty)$ parameter space.

Therefore, the RHS of ($B.2$) is a monotonic decreasing function from infinity to zero on the space $(0,\infty)$ that cuts the FE flat line from above identifying a unique cutoff level φ_D^*. □

Appendix C - Intermediate Economy

Productivity distribution and weighted averages

Let us denote by $\mu_D(\varphi)$, and $\mu_{XI}(\varphi)$ respectively, the productivity distribution of domestic producers, and active innovators and exporters prior to innovation.

$$\mu_D(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_{XI})-G(\varphi_D)} & ,\varphi_{XI} > \varphi \geq \varphi_D \\ 0 & ,otherwise \end{cases}$$

$$\mu_{XI}(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi_{XI})} & ,\varphi \geq \varphi_{XI} \\ 0 & ,otherwise \end{cases}$$

The distributions $\mu_D(\varphi)$, and $\mu_{XI}(\varphi)$ are not affected by the simultaneous entry and exit since the successful entrants and failing incumbents draw their productivity level from the common distribution $\mu(\varphi)$.

Let $\widetilde{\varphi} = \left[\int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)\,d\varphi\right]^{\frac{1}{(\sigma-1)}}$ and $\widetilde{\varphi}_X = \left[\int_{\varphi_{XI}}^{\infty} \varphi^{\sigma-1}\mu_{XI}(\varphi)\,d\varphi\right]^{\frac{1}{(\sigma-1)}}$ denote the average productivity levels of, respectively, all firms and exporting firms only prior to innovation. Then the weighted productivity average that reflects the combined market share of all firms can be defined as

$$\widetilde{\varphi}_t = \left\{\frac{1}{M_t}\left[M\widetilde{\varphi}^{\sigma-1} + nM_X\left(\tau^{-1}\widetilde{\varphi}_X\right)^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}}$$

And let $\widetilde{\varphi}_{XI} = \left[\int_{\varphi_{XI}}^{\infty}\left(\varphi^{\sigma-1}\right)^{\frac{(\alpha+1)}{\alpha}}\mu_{DI}(\varphi)\,d\varphi\right]^{\frac{\alpha}{(\alpha+1)}\frac{1}{(\sigma-1)}}$ represent the average productivity exporter innovators get from innovation.

Aggregate Variables

Denote by m_{XI} and m_D respectively the mass of active innovators and exporters, and non-innovators and non-exporters present in the economy,

$$\begin{aligned} m_{XI} &= \frac{1-G(\varphi_{XI})}{1-G(\varphi_D)}M \\ m_{DI} &= \frac{G(\varphi_{XI})-G(\varphi_{DI})}{1-G(\varphi_D)}M \\ m_D &= \frac{G(\varphi_{DI})-G(\varphi_D)}{1-G(\varphi_D)}M \end{aligned}$$

with M being the mass of incumbent firms in the economy, $M_I = m_{XI}$ the number of firms that perform innovation activities and $M_X = m_{XI}$ the number of firms performing exporting activities. The total number of varieties sold in the economy (by symmetry) will be $M_t = M + nM_X$.
It can be shown that the aggregates will take the following expressions

- Aggregate Price Index

$$P^{1-\sigma} = M_t \left[p_D\left(\widetilde{\varphi}_t\right)\right]^{1-\sigma} + M_I z_D\left(\varphi_{XI}\right)\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\left[p_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)\right]^{1-\sigma}$$

- Aggregate Production

$$Q^{\rho} = M_t \left[q_D\left(\widetilde{\varphi}_t\right)\right]^{\rho} + M_I z_D\left(\varphi_{XI}\right)\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\left[q_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)\right]^{\rho}$$

- Aggregate Revenue

$$R = M_t r_D\left(\widetilde{\varphi}_t\right) + M_I z_D\left(\varphi_{XI}\right)\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} r_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)$$

- Aggregate Profits

$$\begin{aligned}\Pi &= M_t \frac{r_D\left(\widetilde{\varphi}_t\right)}{\sigma} - M f_D - nM_X f_X - M_I f_I \\ &\quad + M_I\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \alpha\left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}}\left(\frac{r_D\left(\widetilde{\varphi}_{XI}\right)}{\sigma}\right)^{\frac{\alpha+1}{\alpha}} \qquad \text{(C.1)}\end{aligned}$$

Intermediate Equilibrium

Proof of Proposition 3, part II

There exist a single cutoff φ^*_{XI} that solves equation (10)

Proof. The proof is divided in three sections

First, I show that the LHS of equation (10) is positive with respect to the productivity parameter. $\pi_{XI}(\varphi_{XI}) - \pi_D(\varphi_{XI}) \geq 0$

$$\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \alpha \left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}} \left[\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}} + n\tau^{1-\sigma}\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1} - nf_X - nf_I \geq 0$$

$$C_1\left(\varphi^{\sigma-1}\right)^{\frac{\alpha+1}{\alpha}} + C_2\varphi^{\sigma-1} - nf_X - f_I \geq 0$$

$$\frac{\partial LHS}{\partial \varphi} = C_1\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)\,\varphi^{\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)-1} + C_2(\sigma-1)\,\varphi^{\sigma-2} > 0$$

Secondly, I show that $\pi_{XI}(\varphi_D) - \pi_D(\varphi_D) < 0$, otherwise the firm would choose to export and innovate instead of being indifferent between innovating or not while staying in the domestic market.

$$\pi_{XI}(\varphi_D) - \pi_D(\varphi_D) < 0$$
$$\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \alpha \left(\frac{f_D}{\alpha+1}\right)^{\frac{\alpha}{\alpha+1}} + n\tau^{1-\sigma} f_D - nf_X - f_I < 0$$

It holds that $\pi_{XI}(\varphi_D) - \pi_D(\varphi_D) < 0$ if:

$$\tau^{\sigma-1} f_X \;>\; +\frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{\left(1+n\tau^{1-\sigma}\right)}$$
$$\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1) + \frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right]}{n\tau^{1-\sigma}} f_I \;>\; \tau^{\sigma-1} f_X$$

□

Proof of Proposition 3, part I

Equations (9) to (10) along with the Free Entry condition (11) completely determine the equilibrium and the productivity cutoffs can be uniquely determined and allow me to rearrange the FE conveniently for the characterizing of the equilibrium as a function of φ_D^*

$$\delta f_E = [1 - G(\varphi_D^*)]\,\overline{\pi}$$

$$\begin{aligned} \delta f_E &= f_D j_1(\varphi_D^*) + n\tau^{1-\sigma} f_D j_2(\varphi_X^*(\varphi_D^*)) - [1 - G(\varphi_{XI}^*)]\, n f_X \\ &\quad + \alpha \left(\frac{f_D}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}} \left(1 + n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} j_3(\varphi_D^*) - [1 - G(\varphi_{XI}^*)]\, f_I \end{aligned} \tag{C.2}$$

where $j_1(\varphi_D^*) = \left[(\widetilde{\varphi}(\varphi_D^*)/\varphi_D^*)^{\sigma-1} - 1\right]$, $j_2(\varphi_D^*) = (\widetilde{\varphi}_x(\varphi_D^*)/\varphi_D^*)^{\sigma-1}[1 - G(\varphi_{XI}^*)]$ and $j_3(\varphi_D^*) = \left[(\widetilde{\varphi}_{XI}(\varphi_D^*)/\varphi_D^*)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}[1 - G(\varphi_{XI}^*)]$

Proof.

Assume the parameter restrictions $\frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]}{n\tau^{1-\sigma}} f_I + \left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1) > \tau^{\sigma-1} f_X$ and $\frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{(1+n\tau^{1-\sigma})} > \tau^{\sigma-1} f_X$ hold, then the Intermediate Equilibrium exists and is unique. I shall proof that the RHS of equation ($C.2$) is decreasing in φ_D^* on the domain (φ_D^*, ∞), so that φ_D^* is uniquely determined by the intersection of the latter curve with the flat line δf_E in the (φ_D^*, ∞) space.
Let $k_1(\varphi_D^*) = \left[(\widetilde{\varphi}(\varphi_D^*)/\varphi_D^*)^{\sigma-1} - 1\right]$,and $k_2(\varphi_D^*) = (\widetilde{\varphi_X}(\varphi_X^*)/\varphi_D^*)^{\sigma-1}$, then

$$k_1'(\varphi_D^*) = \frac{g(\varphi_D^*)}{1 - G(\varphi_D^*)} k_1(\varphi_D^*) - \frac{(\sigma-1)[k_1(\varphi_D^*) + 1]}{\varphi_D^*}$$

$$k_2'(\varphi_D^*) = \frac{g(\varphi_{XI}^*)}{1 - G(\varphi_{XI}^*)}\left[k_2(\varphi_D^*) - \left(\frac{\varphi_{XI}}{\varphi_D}\right)^{\sigma-1}\right] - \frac{(\sigma-1)\, k_2(\varphi_D^*)}{\varphi_D^*}$$

Similarly, $k_3(\varphi_D^*) = \left[(\widetilde{\varphi}_{XI}(\varphi_D^*)/\varphi_D^*)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}$, thus

$$k_3'(\varphi_D^*) = \frac{g(\varphi_{XI}^*)}{1 - G(\varphi_{XI}^*)}\left[k_3(\varphi_D^*) - \left(\frac{\varphi_{XI}^{\sigma-1}}{\varphi_D^{\sigma-1}}\right)^{\frac{\alpha+1}{\alpha}}\right]\frac{\partial \varphi_{XI}^*}{\partial \varphi_D^*} - \left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)\frac{k_3(\varphi_D^*)}{\varphi_D^*}$$

Now, define $j_1(\varphi_D^*) = [1 - G(\varphi_D^*)]\, k_1(\varphi_D^*)$, and $j_2(\varphi_D^*) = [1 - G(\varphi_{XI}^*)]\, k_2(\varphi_D^*)$ and $j_3(\varphi_D^*) = [1 - G(\varphi_{XI}^*)]\, k_3(\varphi_D^*)$ which are non-negative.
Then the derivative and elasticity of $j_1(\varphi_D^*)$,$j_2(\varphi_D^*)$ and $j_3(\varphi_D^*)$ are

$$\frac{j_1'(\varphi_D^*) \cdot \varphi_D^*}{j_1\left(\varphi_D^*\right)} = \underbrace{-(\sigma-1)\left[1 + \frac{1}{k_1\left(\varphi_D^*\right)}\right]}_{<0 \text{ and bounded away of it}} < -(\sigma-1)$$

$$\frac{j_2'(\varphi_D^*) \cdot \varphi_D^*}{j_2\left(\varphi_D^*\right)} = \underbrace{\frac{g(\varphi_{XI})}{1 - G(\varphi_{XI})} \frac{\left(\frac{\varphi_{XI}}{\varphi_D}\right)^{\sigma-1} \frac{\partial \varphi_{XI}}{\partial \varphi_D}}{k_2\left(\varphi_D^*\right)} - (\sigma-1)}_{<0 \text{ and bounded away of it}} < -(\sigma-1)$$

$$\frac{j_3'(\varphi_D^*) \cdot \varphi_D^*}{j_3\left(\varphi_D^*\right)} = \underbrace{-\frac{g\left(\varphi_{DI}^*\right)}{\left[1 - G\left(\varphi_{DI}^*\right)\right]} \frac{\Lambda^{\frac{1}{\sigma-1}} \Lambda^{\frac{\alpha+1}{\alpha}}}{k_2\left(\varphi_D^*\right)} \varphi_D^* - \beta(\sigma-1)}_{<0 \text{ and bounded away of it}} < -\beta(\sigma-1)$$

Thus, $j_1(\varphi_D^*)$, $j_2(\varphi_D^*)$ and $j_3(\varphi_D^*)$ must be decreasing to zero as φ goes to infinite. Furthermore, it must be that $\lim_{\varphi_D^* \to 0} j_1(\varphi_D^*) = \infty$ since $\lim_{\varphi_D^* \to 0} k_1(\varphi_D^*) = \infty$, $\lim_{\varphi_D^* \to 0} j_2(\varphi_D^*) = \infty$ since $\lim_{\varphi_D^* \to 0} k_2(\varphi_D^*) = \infty$ & $\lim_{\varphi_D^* \to 0} j_3(\varphi_D^*) = \infty$ since $\lim_{\varphi_D^* \to 0} k_3(\varphi_D^*) = \infty$. Then $j_1(\varphi_D^*)$, $j_2(\varphi_D^*)$ and $j_3(\varphi_D^*)$, monotonically decrease from infinite to zero on the $(0, \infty)$ parameter space.

Therefore, the RHS of ($C.2$) is a monotonic decreasing function from infinity to zero on the space $(0, \infty)$ that cuts the FE flat line from above identifying a unique cutoff level φ_D^*. □

Appendix D - Aggregates

Aggregate Productivity

In what follows I show that the output of the economy can be expressed as a function of the number of workers in the economy, their productivity and the elasticity of substitution and that equation (15) is the general form of such expression in the open economy. For the proof we use the facts that in equilibrium $L = R$, that the budget constraint is $PQ = R$ and the price rule given by equation (2).

Low Cost Innovation Equilibrium

$$\begin{aligned}
R &= M_t r_D(\widetilde{\varphi}_t) + M_t^I z_D(\varphi_{DI}) \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right) \\
&= M\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} QP^{\sigma}\left\{\int_{\varphi_D}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi\right. \\
&+ z_D(\varphi_{DI})\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\int_{\varphi_{DI}}^{\varphi_{XI}}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi \\
&+ \left. z_D(\varphi_{DI})\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi\right\}
\end{aligned}$$

Then,

$$\begin{aligned}
L &= \left(\frac{\sigma}{\sigma-1}\right) QM^{\frac{1}{1-\sigma}}\left\{\int_{\varphi_D}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi\right. \\
&+ z_D(\varphi_{DI})\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\int_{\varphi_{DI}}^{\varphi_{XI}}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi \\
&+ \left. z_D(\varphi_{DI})\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi\right\}^{\frac{1}{1-\sigma}}
\end{aligned}$$

And

$$Q=\left(\frac{\sigma-1}{\sigma}\right)\left[M\left(\Psi_D+\left(1+n\tau^{1-\sigma}\right)\Psi_X+I^{LCI}\left(\Psi_{DI}+\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\Psi_{XI}\right)\right)\right]^{\frac{1}{\sigma-1}}L \tag{D.1}$$

where $I^{LCI}=z_D\left(\varphi_{DI}\right)\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}$, $\Psi_D=\int_{\varphi_D}^{\varphi_{XI}}\varphi^{(\sigma-1)}\mu(\varphi)d\varphi$, $\Psi_X=\int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)}\mu(\varphi)d\varphi$, $\Psi_{DI}=\int_{\varphi_{DI}}^{\varphi_{XI}}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi$ and $\Psi_{XI}=\int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi$

Intermediate Equilibrium

$$\begin{aligned}
R &= M_t r_D\left(\widetilde{\varphi}_t\right)+m_{XI}\left(1+n\tau^{1-\sigma}\right)z_X\left(\varphi_{XI}\right)\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}r_D\left(\left(\widetilde{\varphi}_{XI}\right)^{\frac{\alpha+1}{\alpha}}\right)\\
&= M\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}QP^{\sigma}\left\{\int_{\varphi_D}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi+n\tau^{1-\sigma}\int_{\varphi_X}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi\right.\\
&+ \left. z_X\left(\varphi_{XI}\right)\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left(1+n\tau^{1-\sigma}\right)\int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi\right\}
\end{aligned}$$

Then,

$$\begin{aligned}
L &= \left(\frac{\sigma}{\sigma-1}\right)QM^{\frac{1}{1-\sigma}}\left\{\int_{\varphi_D}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi+n\tau^{1-\sigma}\int_{\varphi_X}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi\right.\\
&+ \left. z_X\left(\varphi_{XI}\right)\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left(1+\tau^{1-\sigma}\right)\int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi\right\}^{\frac{1}{1-\sigma}}
\end{aligned}$$

And

$$Q=\left(\frac{\sigma-1}{\sigma}\right)\left[M\left(\Psi_D+\left(1+n\tau^{(1-\sigma)}\right)\Psi_X+I^{IE}\left(1+\tau^{1-\sigma}\right)\Psi_{XI}\right)\right]^{\frac{1}{\sigma-1}}L \tag{D.2}$$

where $I^{IE}=z_X\left(\varphi_{XI}\right)\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}$, $\Psi_D=\int_{\varphi_D}^{\varphi_{XI}}\varphi^{(\sigma-1)}\mu(\varphi)d\varphi$, $\Psi_X=\int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)}\mu(\varphi)d\varphi$ and $\Psi_{XI}=\int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi$

Low Cost Trade Equilibrium

$$\begin{aligned} R &= M_t r_D\left(\widetilde{\varphi}_t\right) + m_{XI}\left(1+n\tau^{1-\sigma}\right) z_X(\varphi_{XI}) \left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\widetilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right) \\ &= M\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} QP^{\sigma}\left\{\int_{\varphi_D}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi\right. \\ &+ \left.\left(1+n\tau^{1-\sigma}\right) z_X(\varphi_{XI})\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi\right\} \end{aligned}$$

Then,

$$\begin{aligned} L &= \left(\frac{\sigma}{\sigma-1}\right) QM^{\frac{1}{1-\sigma}}\left\{\int_{\varphi_D}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi\right. \\ &+ \left.\left(1+n\tau^{1-\sigma}\right) z_X(\varphi_{XI})\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi\right\}^{\frac{1}{1-\sigma}} \end{aligned}$$

And

$$Q = \left(\frac{\sigma-1}{\sigma}\right)\left[M\left(\Psi_D + (1+n\tau^{1-\sigma})\Psi_X + I^{LCT}\left(1+n\tau^{1-\sigma}\right)\Psi_{XI}\right)\right]^{\frac{1}{\sigma-1}} L \qquad \text{(D.3)}$$

where $I^{LCT} = z_X(\varphi_{XI})\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}$,$\Psi_D = \int_{\varphi_D}^{\varphi_X}\varphi^{(\sigma-1)}\mu(\varphi)d\varphi$, $\Psi_X = \int_{\varphi_X}^{\infty}\varphi^{(\sigma-1)}\mu(\varphi)d\varphi$ and $\Psi_{XI} = \int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi$

Proof of Proposition 4

Low Cost Innovation Equilibrium

$$\Psi^{LCI} = \left[M\left(\Psi_D + \left(1+n\tau^{1-\sigma}\right)\Psi_X + I^{LCI}\left(\Psi_{DI} + \left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\Psi_{XI}\right)\right)\right]^{\frac{1}{\sigma-1}}$$

$$
\begin{aligned}
\Delta log \Psi^{LCI} \quad &= \quad \underbrace{-s_X \Delta log(\tau)}_{Exports} \underbrace{- \left(\frac{\alpha+1}{\alpha}\right)\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right) s_{XI} I^{LCI} \Delta log(\tau)}_{Exporters'\ Innovation} \\
&+ \quad \frac{1}{\sigma-1}\left[\underbrace{\Delta log(M)}_{Entry\ Effect} + \underbrace{s_D \Delta log\left(\Psi_D\right)}_{Domestic\ Market} + \underbrace{\left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right) s_X \Delta log\left(\Psi_X\right)}_{Export\ Market}\right. \\
&+ \quad \left.\underbrace{s_I I^{LCI} \Delta log(I^{LCI})}_{Extensive\ Margin} + \underbrace{I^{LCI}\left[\left(s_I - s_{XI}\right) \Delta log(\Psi_{DI}) \Delta + s_{XI} log(\Psi_{XI})\right]}_{Intensive\ Margin}\right]
\end{aligned}
$$

Proof. Recall that for every $x \in \mathcal{R}$

$$
\frac{\Delta x}{x} = \Delta log(x)
$$

Take logs of Ψ^{LCI}

$$
log\left(\Psi^{LCI}\right) = \tfrac{1}{\sigma-1}\left[log(M) + log\left(\Psi_D + \left(1+n\tau^{1-\sigma}\right)\Psi_X + I^{LCI}\left(\Psi_{DI} + \left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \Psi_{XI}\right)\right)\right]
$$

And derivatives

$$
\begin{aligned}
&\Delta log \Psi = \frac{1}{\sigma-1}\left[\Delta log(M) + \Delta log \hat{\Psi}\right] \\
&\Delta log \hat{\Psi} = \frac{1}{\hat{\Psi}}\left[\Delta \Psi_D + \Delta\left(1+\tau^{1-\sigma}\right)\Psi_X + \left(1+\tau^{1-\sigma}\right)\Delta\Psi_X\right. \\
&+ \Delta\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\right] I\Psi_I + \left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \Delta I \Psi_I + \left.\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} I \Delta\Psi_I\right]
\end{aligned}
$$

Define the share of domestic firms excluding innovation $s_D = \frac{\Psi_D}{\hat{\Psi}}$, the share of export firms excluding innovation $s_X = \frac{n\tau^{1-\sigma}\Psi_X}{\hat{\Psi}}$, the share of innovation activities $s_I = \frac{\Psi_{INN}}{\hat{\Psi}}$ and the share of exporters innovation activities $s_{XI} = \frac{\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\Psi_{XI}}{\hat{\Psi}}$. Then, the variation in productivity can be decomposed in the following terms:

- Direct Effect on Exports $= -s_X \Delta log(\tau)$
- Direct Effect on Exporters' Innovation $= -\left(\frac{\alpha+1}{\alpha}\right)\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right) s_{XI} I^{LCI} \Delta log(\tau)$
- Indirect Entry Effect $= \left(\frac{1}{\sigma-1}\right) \Delta log(M)$
- Indirect Domestic Market Effect $= \left(\frac{1}{\sigma-1}\right) s_D \Delta log\left(\Psi_D\right)$
- Indirect Export Market Effect $= \left(\frac{1}{\sigma-1}\right)\left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right) s_X \Delta log\left(\Psi_X\right)$
- Indirect Extensive Margin Innovation Effect $= \left(\frac{1}{\sigma-1}\right) s_I I^{LCI} \Delta log(I^{LCI})$
- Indirect Intensive Margin Innovation Effect $= \left(\frac{1}{\sigma-1}\right) I^{LCI}\left(s_I - s_{XI}\right) \Delta log(\Psi_{DI}) + \left(\frac{1}{\sigma-1}\right) I^{LCI} s_{XI} \Delta log$

□

Intermediate Equilibrium

$$\Psi^{IE} = \left[M\left(\Psi_D + \left(1 + n\tau^{(1-\sigma)}\right)\Psi_X + I^{IE}\left(1 + n\tau^{1-\sigma}\right)\Psi_{XI}\right)\right]^{\frac{1}{\sigma-1}}$$

$$\begin{aligned}
\Delta log \Psi^{IE} \quad = \quad & \underbrace{-s_X \Delta log(\tau)}_{Exports} \underbrace{- \left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right) s_{XI} I^{IE} \Delta log(\tau)}_{Exporters'\ Innovation} \\
+ \quad & \frac{1}{\sigma-1}\left[\underbrace{\Delta log(M)}_{Entry\ Effect} + \underbrace{s_D \Delta log\left(\Psi_D\right)}_{Domestic\ Market} + \underbrace{\left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right) s_X \Delta log\left(\Psi_X\right)}_{Export\ Market}\right. \\
+ \quad & \left.\underbrace{s_{XI} I^{IE} \Delta log(I^{IE})}_{Extensive\ Margin} + \underbrace{I^{IE} s_{XI} log(\Psi_I)}_{Intensive\ Margin}\right]
\end{aligned}$$

Proof. Recall that for every $x \in \mathcal{R}$

$$\frac{\Delta x}{x} = \Delta log(x)$$

Take logs of Ψ^{IE}

$$log\left(\Psi^{IE}\right)=\frac{1}{\sigma-1}\left[log(M)+log\left(\Psi_D+\left(1+n\tau^{1-\sigma}\right)\Psi_X+I^{IE}\left(1+n\tau^{1-\sigma}\right)\Psi_I\right)\right]$$

And derivatives

$$\begin{aligned}\Delta log\Psi &= \frac{1}{\sigma-1}\left[\Delta log(M)+\Delta log\hat{\Psi}\right]\\ \Delta log\hat{\Psi} &= \frac{1}{\hat{\Psi}}\left[\Delta\Psi_D+\Delta\left(1+n\tau^{1-\sigma}\right)\Psi_X+\left(1+n\tau^{1-\sigma}\right)\Delta\Psi_X\right.\\ &+ \left.\Delta\left(1+n\tau^{1-\sigma}\right)I\Psi_I+\left(1+n\tau^{1-\sigma}\right)\Delta I\Psi_I+\left(1+n\tau^{1-\sigma}\right)I\Delta\Psi_I\right]\end{aligned}$$

Define the share of domestic firms excluding innovation $s_D=\frac{\Psi_D}{\hat{\Psi}}$, the share of export firms excluding innovation $s_X=\frac{n\tau^{1-\sigma}\Psi_X}{\hat{\Psi}}$ and the share of exporters innovation activities $s_{XI}=\frac{\left(1+n\tau^{1-\sigma}\right)\Psi_I}{\hat{\Psi}}$. Then, the variation in productivity can be decomposed in the following terms:

- Direct Effect on Exports $=-s_X\Delta log(\tau)$
- Direct Effect on Exporters' Innovation $=-\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right)s_{XI}I^{IE}\Delta log(\tau)$
- Indirect Entry Effect $=\left(\frac{1}{\sigma-1}\right)\Delta log(M)$
- Indirect Domestic Market Effect $=\left(\frac{1}{\sigma-1}\right)s_D\Delta log\left(\Psi_D\right)$
- Indirect Export Market Effect $=\left(\frac{1}{\sigma-1}\right)\left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right)s_X\Delta log\left(\Psi_X\right)$
- Indirect Extensive Margin Innovation Effect $=\left(\frac{1}{\sigma-1}\right)s_{XI}I^{IE}\Delta log(I^{IE})$
- Indirect Intensive Margin Innovation Effect $=\left(\frac{1}{\sigma-1}\right)s_{XI}I^{IE}\Delta log(\Psi_{XI})$

□

Low Cost Trade Equilibrium

$$\Psi^{LCT}=\left[M\left(\Psi_D+(1+n\tau^{1-\sigma})\Psi_X+I^{LCT}\left(1+n\tau^{1-\sigma}\right)\Psi_{XI}\right)\right]^{\frac{1}{\sigma-1}}$$

$$
\begin{aligned}
\Delta log\Psi^{LCT} &= \underbrace{-s_X\Delta log(\tau)}_{Exports}\underbrace{-s_{XI}I^{LCT}\Delta log(\tau)}_{Exporters'\ Innovation} \\
&+ \frac{1}{\sigma-1}\Bigg[\underbrace{\Delta log(M)}_{Entry\ Effect} + \underbrace{s_D\Delta log\left(\Psi_D\right)}_{Domestic\ Market} + \underbrace{\left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right)s_X\Delta log\left(\Psi_X\right)}_{Export\ Market} \\
&+ \underbrace{\left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right)s_{XI}I^{LCT}\Delta log(I^{LCT})}_{Extensive\ Margin} + \underbrace{\left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right)s_{XI}I^{LCT}log(\Psi_I)}_{Intensive\ Margin}\Bigg]
\end{aligned}
$$

Proof. Recall that for every $x \in \mathcal{R}$

$$\frac{\Delta x}{x} = \Delta log(x)$$

Take logs of Ψ^{LCT}

$$log\left(\Psi^{LCT}\right) = \frac{1}{\sigma-1}\left[log(M) + log\left(\Psi_D + \left(1+n\tau^{1-\sigma}\right)\Psi_X + I^{LCT}\left(1+n\tau^{1-\sigma}\right)\Psi_I\right)\right]$$

And derivatives

$$
\begin{aligned}
\Delta log\Psi &= \frac{1}{\sigma-1}\left[\Delta log(M) + \Delta log\hat{\Psi}\right] \\
\Delta log\hat{\Psi} &= \frac{1}{\hat{\Psi}}\left[\Delta\Psi_D + \Delta\left(1+n\tau^{1-\sigma}\right)\Psi_X + \left(1+n\tau^{1-\sigma}\right)\Delta\Psi_X\right. \\
&+ \left.\Delta\left(1+n\tau^{1-\sigma}\right)I\Psi_I + \left(1+n\tau^{1-\sigma}\right)\Delta I\Psi_I + \left(1+n\tau^{1-\sigma}\right)I\Delta\Psi_I\right]
\end{aligned}
$$

Define the share of domestic firms excluding innovation $s_D = \frac{\Psi_D}{\hat{\Psi}}$, the share of export firms excluding innovation $s_X = \frac{n\tau^{1-\sigma}\Psi_X}{\hat{\Psi}}$ and the share of exporters innovation activities $s_{XI} = \frac{\left(1+n\tau^{1-\sigma}\right)\Psi_{XI}}{\hat{\Psi}}$. Then, the variation in productivity can be decomposed in the following terms:

- Direct Effect on Exports $= -s_X\Delta log(\tau)$

- Direct Effect on Exporters' Innovation $= -\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right) s_{XI} I^{LCT} \Delta log(\tau)$
- Indirect Entry Effect $= \left(\frac{1}{\sigma-1}\right) \Delta log(M)$
- Indirect Domestic Market Effect $= \left(\frac{1}{\sigma-1}\right) s_D \Delta log\left(\Psi_D\right)$
- Indirect Export Market Effect $= \left(\frac{1}{\sigma-1}\right) \left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right) s_X \Delta log\left(\Psi_X\right)$
- Indirect Extensive Margin Innovation Effect $= \left(\frac{1}{\sigma-1}\right) s_{XI} I^{LCT} \Delta log(I^{LCT})$
- Indirect Intensive Margin Innovation Effect $= \left(\frac{1}{\sigma-1}\right) s_{XI} I^{LCT} \Delta log(\Psi_{XI})$

□