Aging, social security design and capital accumulation

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December 4, 2013

Abstract

The purpose of this paper is to analyze the impact of aging on capital accumulation and welfare in a country that exhibits a sizable and unfunded social security system. We use a two-overlapping-generation model of growth with endogenous retirement decision. We consider two demographic changes : declining fertility and increasing longevity and three types of pensions : defined contributions, defined benefits and defined annuities. Both types of aging and social security have specific impacts on capital accumulation and welfare.

Keywords: aging, public finance sustainability, social security

JEL Classification: H2, F42, H8

1 Introduction

One of the main challenges of industrialized economies is demographic aging. Demographic aging that results from two concomitant factors, an ever increasing longevity and a sharp decline in fertility, implies a raise in the ratio of elderly to the rest of population, the so called dependency rate. This evolution is expected to have a number of consequences, which are not always clear. Some are clearly unfavorable; this is the case of the effect of increasing dependency on the financial balance of unfunded pensions. Some are perceived as positive; if the decline in fertility outweighs the increase in longevity we would have a population decrease that some environmentalists may welcome. Finally there are some ambiguous effects. One of them is that of aging on capital accumulation, which is one the key factors of growth. Studying the effect of aging on capital accumulation is made particularly difficult in a setting where the public sector is important and dependent on the age structure, which is the case of national debt, long term care and unfunded social security.

In this paper, we use a two period overlapping generation model, which enables us to assess the level of capital accumulation and the sustainability of fiscal policy in a society subject to two demographic changes: fertility decline and increased longevity. Those two changes contribute to a drastic aging of society namely an increase in the ratio of old to young people and thus an unbearable pressure on the sustainability of public finances.

Relative to the canonical Diamond's model, our model will comprise a number of features that make it closer to the real world and can lend itself to calibration.

- Endogenous age of retirement.
- Pay-as-you go social security that can be either defined benefits, defined contributions or defined annuitites.

The main intended contribution of this study is to analyze the incidence of these two factors of aging on capital accumulation and social welfare. We will see that this incidence will depend on the type of social security (funded or not, defined benefits or defined contributions), and on the way the retirement decision is regulated. The rest of the paper is organized as follows. In section 2 we present the basic model and the main results for an economy consisting of identical individuals and having a defined contribution pension system. Section 3 is devoted to a comparison of defined benefits and defined contributions pensions systems . A fourth section is devoted to the dynamics of this model. As it appears clearly the demographic shock will differ according to the type of social security and according to the sense of aging.

2 The basic model

We use the standard overlapping generation model. An individual belonging to generation t lives two periods t and $t+1$. The first one has a unitary length, while the second has a length $\ell \leq 1$, where ℓ reflects variable longevity. In the first period, the individual works and earns w_t which is devoted to the first-period consumption, c_t, saving s_t and pension contribution τ. In the second period he works an amount of time $z_{t+1} \leq \ell \leq 1$ and earns $z_{t+1}w_{t+1}$. This earning plus the proceeds of saving $R_{t+1}s_t$ and the PAYG pension p finances second period consumption d_{t+1}. Working z_{t+1} implies a monetary disutility $v(z_{t+1}, \ell)$ where $\frac{\partial v}{\partial \ell} < 0$ reflects the idea that an increase in longevity fosters later retirement. Note that in this simple model we assume for simplicity that earnings in the second period of life is not taxed, this is for the sake of simplicity. It could be justified by stating that the end of the first period coincides with the statutory age of retirement; also we assume that a fully funded system is identical to standard saving. Thus the parameter τ measures the relative size of the unfunded pensions. In other words $\tau = 0$ implies that the whole pension system is funded.

Denoting by $u(\cdot)$ the utility function for consumption c or d and U the lifetime utility, the problem of an individual of generation t is:

$$\max U_t = u\left(w_t - \tau - s_t\right) + \beta \ell u \left(\frac{w_{t+1} z_{t+1} + R_{t+1} s_t + p - v\left(z_{t+1}, \ell\right)}{\ell} \right) \tag{1}$$

where $p = \tau(1+n)$ and β is the time discount factor. $(1+n)$ is the gross rate of population growth and also the number of children per individual.

The FOC's are simply:

$$\begin{aligned} & v'_{z_{t+1}}\left(z_{t+1}, \ell\right) = w_{t+1} \\ & \beta R_{t+1} u'\left(\tilde{d}_{t+1}\right) - u'\left(c_t\right) = 0 \end{aligned}$$

where $\tilde{d}_{t+1} = d_{t+1} - v(z_{t+1}, \ell)$. Again for simplicity's sake, we will use simple forms for $u(\cdot)$ and $v(\cdot)$: $u(x) = \ln x$ and $v(x) = x^2/2\gamma\ell$. One clearly sees that the disutility of working longer is mitigated by an increase in longevity. We can now write the problem of the individual as the following maximization of:

$$U_t = \ln\left(w_t - \tau - s_t\right) + \beta \ell \ln \left(\frac{R_{t+1} s_t + z^2/2\gamma\ell + p}{\ell} \right) \tag{2}$$

where $p_t = \tau(1+n)$[1]. The FOC with respect to z_{t+1} and s_t yield :

$$z_{t+1} = z^*_{t+1} = \gamma \ell w_{t+1} \tag{3}$$

$$s_t = \frac{\beta\ell}{1+\beta\ell} w_t - \frac{\gamma \ell w_{t+1}^2}{2R_{t+1}\left(1+\beta\ell\right)} - \tau \left(\frac{\beta\ell}{1+\beta\ell} + \frac{1+n}{\left(1+\beta\ell\right) R_{t+1}} \right) \tag{4}$$

In many countries, z is not the outcome of a choice without distortion. Through an array of programs, workers are induced to retire earlier than they would choose

[1] As seen below, this implies defined contributions.

to do without these programs. We will denote this induced early retirement by $\bar{z}$[2]. In the case of early retirement, we rewrite equations (3) and (4) as follows :

$$z_{t+1} = \bar{z}$$

$$s_t = \frac{\beta\ell}{1+\beta\ell}w_t - \frac{\bar{z}}{R_{t+1}\left(1+\beta\ell\right)}\left(w_{t+1} - \bar{z}/2\gamma\ell\right) - \tau\left(\frac{\beta\ell}{1+\beta\ell} + \frac{1+n}{\left(1+\beta\ell\right)R_{t+1}}\right) \quad (2)$$

We now turn to the production side. We use a Cobb-Douglas production function :

$$Y_t = F\left(K_t, N_t\right) = AK_t^{\alpha}N_t^{1-\alpha} \quad (3)$$

where the labor force is $N_t = L_t + L_{t-1}z_t = L_{t-1}\left(1+n+z_t\right)$, K is the stock of capital and A is a productivity parameter. We distinguish N_t the labor force and L_t the size of generation t. We assume that :

$$L_t = L_{t-1}\left(1+n\right).$$

Total population at time t is :

$$L_t + \ell L_{t-1} = L_{t-1}\left(1+\ell+n\right).$$

Denoting $K_t/N_t \equiv k_t$ and $Y_t/N_t \equiv y_t$, we obtain the income per worker (and not per capita):

$$y_t = f\left(k_t\right) = Ak_t^{\alpha}$$

[2] An alternative specification could be that second period labor is subject to a proportional tax θ whose proceeds are returned to the old workers. Their problem would be to choose z such that :

$$wz(1-\theta) + T - v(z,\ell)$$

with $T = \theta wz$, $v = z^2/2\gamma\ell$, this yields

$$z = \gamma\ell w(1-\theta)$$

In the case of optimal distortionless retirement,

$$z = z^* = \gamma\ell w$$

In the case of induced early retirement,

$$z = \bar{z} = \gamma\ell w(1-\theta)$$

where θ is chosen such as to generate $\bar{z} < z^*$

and the factor prices :

$$R_t = f'(k_t) = A\alpha k_t^{\alpha-1}$$
$$w_t = f(k_t) - f'(k_t)k_t = (1-\alpha) A k_t^{\alpha}$$

while the equilibrium conditions in the labor and capital markets are respectively :

$$N_t = L_{t-1}(1+n+z_t)$$
$$K_{t+1} = L_t s_t$$

The latter expression can be rewritten as follows :

$$\begin{aligned} G_t = (1+n+z_{t+1})\,k_{t+1} - \frac{\beta\ell}{1+\beta\ell}A(1-\alpha)k_t^{\alpha} + \tau\left(\frac{\beta\ell}{1+\beta\ell} + \frac{(1+n)\,k_{t+1}^{1-\alpha}}{A\alpha\,(1+\beta\ell)}\right) \\ + \frac{z_{t+1}k_{t+1}^{1-\alpha}}{(1+\beta\ell)\,A\alpha}\left(A(1-\alpha)k_{t+1}^{\alpha} - \frac{z_{t+1}}{2\gamma\ell}\right) = 0 \end{aligned} \tag{4}$$

The standard (Diamond) case with $z = \tau = 0$ reduces to :

$$G_t = (1+n+z_{t+1})\,k_{t+1} - \frac{\beta\ell}{1+\beta\ell}A(1-\alpha)k_t^{\alpha} \tag{5}$$

Comparing (4) and (5) we have two main differences :

- The third term of the RHS of (4), which includes the double burden that the PAYG imposes to saving.
- The fourth term, which reflects the double effect of working in the second period: a distortionary effect if z is not optimal and a saving inducement if $z < z^*$

In equation (4) and (2) we assumed a pension system that relies on a defined contribution (DC) formula, in which the tax τ is given and thus the benefit p has to follow through.Two alternative systems can also be considered.The first one provides a defined benefit (DB) p over the second period: the contribution rate is then endogenous. The other is a scheme which offers constant annuities (DA) during retirement. For the clarity's sake, let us present the three revenue constraints that these formulas imply (we use an upper bar for the defined variable) :

$$DC : \bar{\tau}(1+n) = p$$
$$DB : \tau(1+n) = \bar{p}$$
$$DA : \bar{a}(\ell - z) = \tau(1+n),$$

Table 1: 6 types of social security regimes

	DC	DB	DC
$z = \bar{z} < z^*$	1	2	3
z^*	4	5	6

$z^* = \gamma \ell w.$

where $\bar{a}$ is the defined annuity and τ has to adjust to variations in z, ℓ and n. Note that for each type of pension the individual utility has to adjust accordingly. We have thus two characteristics for a social security system : is it DB, DC, DA and does it comprises an early age or optimal age of retirement ? Equation (4) can take 6 values depending on the type of social security regime that prevails. The six cases are presented on Table 1, and the different equations $G_{i,t}$ are provided in appendix.

Using the 6 equations given in the appendix, we now turn to the comparative statics of the problem.

3 Comparative Statics

We start by the case of early retirement.

$$\Delta \frac{\partial k^1}{\partial n} = -A\alpha(1+\beta\ell)k - \bar{\tau}k^{1-\alpha} < 0 \tag{6}$$

$$\Delta \frac{\partial k^2}{\partial n} = -A\alpha(1+\beta\ell)k + \frac{\bar{p}A\alpha k^{1-\alpha}}{(1+n)^2} \gtrless 0 \tag{7}$$

$$\Delta \frac{\partial k^3}{\partial n} = -A\alpha(1+\beta\ell)k + \frac{\bar{a}(\ell - z)A\alpha k^{1-\alpha}}{(1+n)^2} \gtrless 0 \tag{8}$$

Where $\Delta = \left(\frac{\partial \tilde{G}}{\partial k}\right)^{-1} > 0$ and the superscript denotes the type of social security. An increase in fertility has a depressive effect in the absence of pension. This effect is reinforced with a DC pension, but weakened or even reversed with DB or DA pensions.

$$\Delta\frac{\partial k^1}{\partial \ell} = A\alpha[A\beta(1-\alpha)k^\alpha - (1+n+\bar{z})k] - \frac{\bar{z}^2k^{1-\alpha}}{2\gamma\ell^2} - \bar{\tau}A\alpha\beta \gtreqless 0 \tag{9}$$

$$\Delta\frac{\partial k^2}{\partial \ell} = A\alpha[A\beta(1-\alpha)k^\alpha - (1+n+\bar{z})k] - \frac{\bar{z}^2k^{1-\alpha}}{2\gamma\ell} - \frac{\bar{p}A\alpha\beta}{1+n} \gtreqless 0 \tag{10}$$

$$\begin{aligned}\Delta\frac{\partial k^3}{\partial \ell} &= A\alpha[A\beta(1-\alpha)k^\alpha - (1+n+\bar{z})k] - \frac{\bar{z}^2k^{1-\alpha}}{2\gamma\ell} \\ &- \bar{a}\left[\left(\frac{A\alpha\beta\ell}{1+n} + k^{1-\alpha}\right) + \frac{A\alpha\beta}{1+n}(\ell-\bar{z})\right] \gtreqless 0\end{aligned} \tag{11}$$

An increase in longevity has a fostering effect on capital accumulation without pension and work in the second period (term in brackets). When these features are introduced, this effect is diminished and could be reversed, particularly in the case of defined annuities. An increased longevity increases the cost of mandatory retirement (2nd term) and the cost of the pension system (3rd term).

We now turn to the comparative statics when retirement is chosen optimally. Compared to the case with early retirement, we have less distortion but also less incentives for saving.

$$\Delta\frac{\partial k^4}{\partial n} = -A\alpha(\ell^{-1}+\beta)k - \bar{\tau}\ell^{-1}k^{1-\alpha} < 0 \tag{12}$$

$$\Delta\frac{\partial k^5}{\partial n} = -A\alpha(\ell^{-1}+\beta)k + \frac{\bar{p}A\alpha\beta}{1+n} \gtreqless 0 \tag{13}$$

$$\Delta\frac{\partial k^6}{\partial n} = -A\alpha(\ell^{-1}+\beta)k + \frac{\bar{a}A\alpha\beta\ell(1-\gamma D)}{(1+n)^2} \gtreqless 0 \tag{14}$$

As above, the pension system reinforces the depressive effect of fertility on capital accumulation in the DC case and weakens it in the DB or DA case. Turning to the effect of longevity when z is endogeneous, we have:

$$\Delta\frac{\partial k^4}{\partial \ell} = A\alpha\ell^{-2}(1+n)k - A^2\alpha\gamma\beta(1-\alpha)k^{1+\alpha} + \bar{\tau}\ell^{-2}(1+n)k^{1-\alpha} \gtreqless 0 \tag{15}$$

$$\Delta\frac{\partial k^5}{\partial \ell} = A\alpha\ell^{-2}(1+n)k - A^2\alpha\gamma\beta(1-\alpha)k^{1+\alpha} + \bar{p}\ell^{-2}k^{1-\alpha} \gtreqless 0 \tag{16}$$

$$\begin{aligned}\Delta\frac{\partial k^6}{\partial \ell} &= A\alpha\beta[A\alpha\beta(1-\alpha)k^\alpha - (1+n)k] - A\gamma Dk(1+\alpha\beta\ell) \\ &- \frac{2A\alpha\beta\ell\bar{a}(1-D\gamma)}{1+n} + \frac{\gamma D^2k^{1-\alpha}}{2} - \bar{a}(1-\gamma D)k^{1-\alpha} \gtreqless 0\end{aligned} \tag{17}$$

where $D = (A(1-\alpha)k^\alpha - \bar{a})$

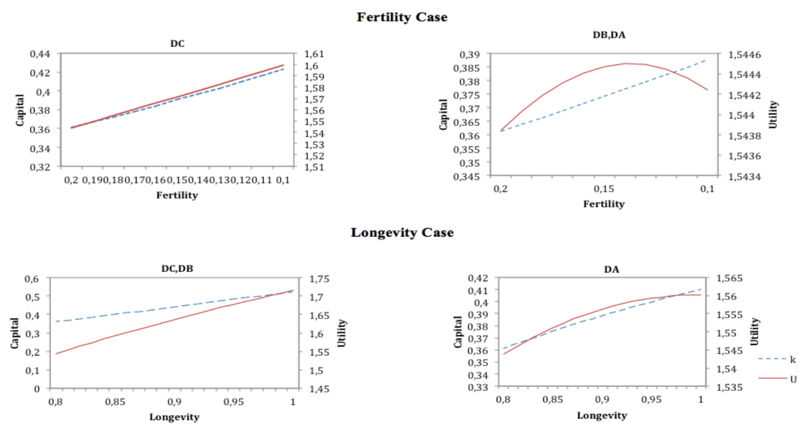
One sees that with both DC and DB, the pension contributes to the positive effect on capital accumulation whereas the retirement term goes into the opposite direction (discouraging saving). With DA, both pension and retirement weaken the positive effect of ℓ and k.

We now turn to a numerical example to better grasp the effect of n and ℓ on both k and U in the steady state. To do that we use the following parameters values : With these parameters we can obtain the profils of U and k as n decreases or ℓ

$A = 10$	$\alpha = 0.33$	$\gamma = 0.05$	$z = 0.05$
$\beta = 0.25$	$n = 0.2$	$\ell = 0.6$	$\tau = 1$

increases. They are presented on the following figures.

Figure 1: SS utility and capital: Early Retirement



Our numerical illustration indicates that in most cases the incidence of aging has the same sign as without pension or activity in the second period.
There are exceptions. We start with the case of early retirement illustrated in Figure 1. An increase in fertility may has a positive effect on utility for low rates of fertility in case of defined benefits or defined annuities. This means that the decline in capital accumulation is more than offset by the fact that the pension burden is alleviated by an increase in fertility. As to longevity increase, we see that in the case of DA it can have a depressive effect on welfare when longevity is high enough. The reason is to be found in the fact that the age of retirement increases as well, which induces a loss in utility. Turning to the case of optimal retirement, we again find that utility increases as n increases under DB or DA regimes. When

Figure 2: Optimal Retirement 1

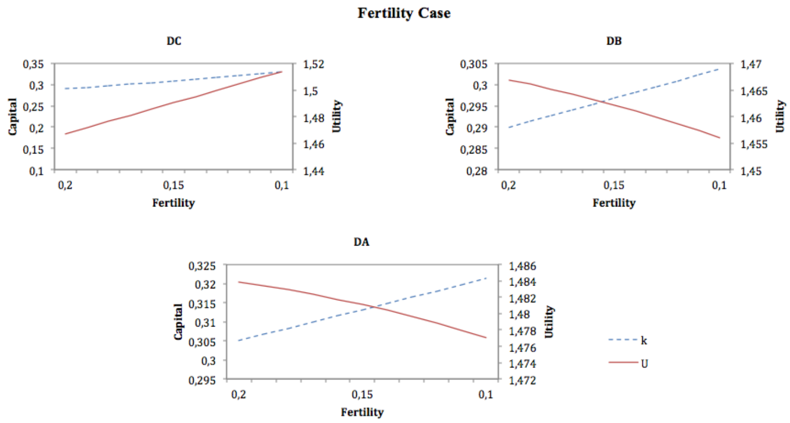
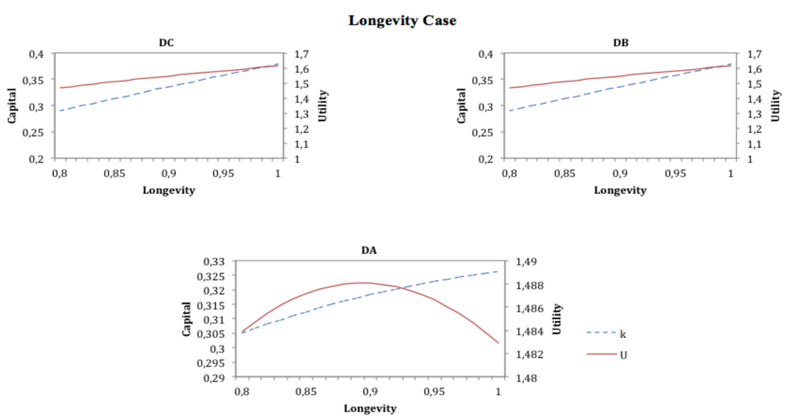


Figure 3: Optimal Retirement 2



longevity increases and social security is of the DA type, utility decreases for high levels of longevity. People living longer later implies more disutility of work and less consumption in the second period.

4 Dynamics

We now turn to the dynamics of the problem. We are particularly interested by the gains (if any) in capital accumulation and in utility resulting from aging and by the cases where the transition can be welfare worsening for some generations. These results are given in Table 2 and are illustrated by the Figures 4 and 5. To make comparable the changes in n and ℓ, we consider a change of n from 0.2 to 0.1 and a change of ℓ from 0.8 to 0.873. Both imply the same increase in the rate of dependency $\frac{\ell}{1+n}$.

Table 2: Comparative Statics for Capital per Worker and Lifetime Utility under Different Social Security Systems and Shock Scenarios
(Percentage Change in the Steady State Values after the shock)

	Longevity Shock		Fertility Shock	
	Capital per Worker	Lifetime Utility	Capital per Worker	Lifetime Utility
Early Retirement				
Defined Contribution	16.22	4.64	17.37	3.79
Defined Benefit	16.22	4.64	7.31	0.03
Defined Annuity	5.59	0.63	7.31	0.03
Optimal Retirement				
Defined Contribution	11.75	4.17	13.87	3.37
Defined Benefit	11.75	4.17	4.73	-0.76
Defined Annuity	3.72	0.40	5.88	-0.27

The most striking result is the loss in utility following a fertility shock, particularly with DC.This is due to substantial loss in second period consumption. From Table 2 and Figures 4-5, it appears that the defined contribution formula dominates the other formulas both in capital increase and utility gain in the steady state. Induced early retirement seems also to dominate flexible retirement meaning that the labor distortion is more tha offset by the gain in capital accumulation.

Focusing on the transition, one observes a one generation drop in utility with defined contribution when there is a longevity shock.

Figure 4: Early Retirement

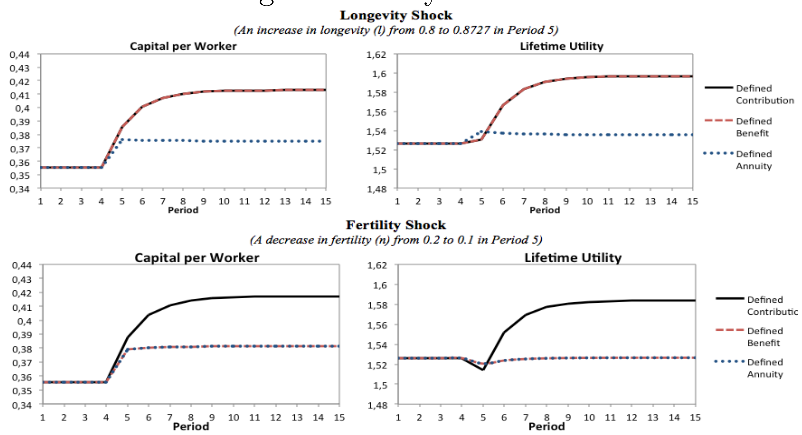
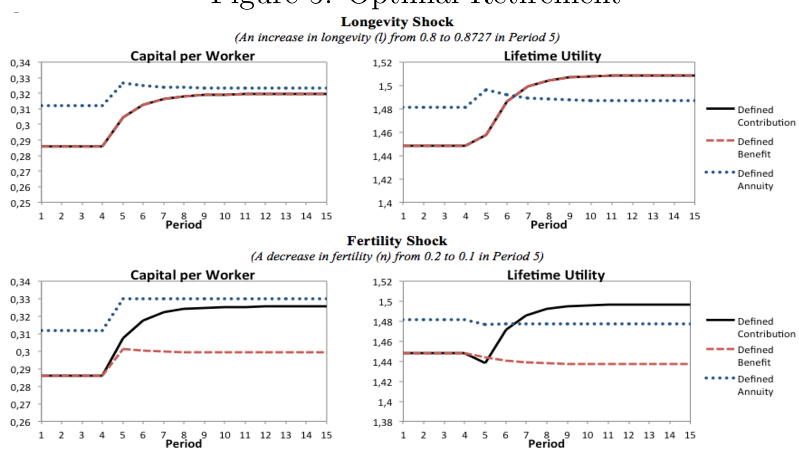


Figure 5: Optimal Retirement



5 Conclusions

In this paper we have tried to evaluate the implications of different sources of aging on both the level of capital and of welfare of economies having social security systems that can differ in three ways: defined benefits or contributions, funded or not, early retirement or flexible retirement. We show that the effects of longevity increase and fertility decrease, two phenomena that contribute to aging, vary in a contrasted way depending on these features of the pension system. This is important as we know that countries do not age the same way and that they tend to shift progressively from a regime of defined benefits towards one of defined contributions. On our further research agenda, we intend to look at the joint effect of aging and changes in the social security regimes: from DA/DB to DC and from early retirement to flexible retirement. Another limitation of the the current analysis is that individuals are all identical. With heterogeneity in wages, and would find more merits in the defined annuity formula.

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Appendix

$$G = (1+n+z_{t+1})k_{t+1} - \frac{\beta\ell}{1+\beta\ell}A(1-\alpha)k_t^{\alpha} + \tau\left(\frac{\beta\ell}{1+\beta\ell} + \frac{(1+n)k_{t+1}^{1-\alpha}}{A\alpha(1+\beta\ell)}\right) \tag{18}$$

$$+ \frac{z_{t+1}k_{t+1}^{1-\alpha}}{A\alpha(1+\beta\ell)}(A(1-\alpha)k_{t+1}^{\alpha} - z_{t+1}/2\gamma\ell)$$

$$\tilde{G}_1 = G_1(A\alpha(1+\beta\ell)) = A\alpha(1+\beta\ell)(1+n+\bar{z})k + \bar{z}A(1-\alpha)k - \frac{\bar{z}^2k^{1-\alpha}}{2\gamma\ell} \tag{19}$$

$$+ \bar{\tau}\left(A\alpha\beta\ell + (1+n)k^{1-\alpha}\right) - A^2\alpha\beta\ell(1-\alpha)k^{\alpha}$$

$$\tilde{G}_2 = G_2(A\alpha(1+\beta\ell)) = A\alpha(1+\beta\ell)(1+n+\bar{z})k + \bar{z}A(1-\alpha)k - \frac{\bar{z}^2k^{1-\alpha}}{2\gamma\ell} \tag{20}$$

$$+ \bar{p}\left(\frac{A\alpha\beta\ell}{1+n} + k^{1-\alpha}\right) - A^2\alpha\beta\ell(1-\alpha)k^{\alpha}$$

$$\tilde{G}_3 = G_3(A\alpha(1+\beta\ell)) = A\alpha(1+\beta\ell)(1+n+\bar{z})k + \bar{z}A(1-\alpha)k - \frac{\bar{z}^2k^{1-\alpha}}{2\gamma\ell} \tag{21}$$

$$+ \bar{a}(\ell-\bar{z})\left(\frac{A\alpha\beta\ell}{1+n} + k^{1-\alpha}\right) - A^2\alpha\beta\ell(1-\alpha)k^{\alpha}$$

$$\tilde{G}_4 = G_4\frac{(A\alpha(1+\beta\ell))}{\ell} = A\alpha(\ell^{-1}+\beta)(1+n)k + A^2\alpha\gamma(1+\beta\ell)(1-\alpha)k^{1+\alpha} \tag{22}$$

$$+ \frac{A^2\gamma(1-\alpha)^2k^{1+\alpha}}{2} + \bar{\tau}\left(A\alpha\beta + (1+n)\ell^{-1}k^{1-\alpha}\right) - A^2\alpha\beta(1-\alpha)k^{\alpha}$$

$$\tilde{G}_5 = G_5\frac{(A\alpha(1+\beta\ell))}{\ell} = A\alpha(\ell^{-1}+\beta)(1+n)k + A^2\alpha\gamma(1+\beta\ell)(1-\alpha)k^{1+\alpha} \tag{23}$$

$$+ \frac{A^2\gamma(1-\alpha)^2k^{1+\alpha}}{2} + \bar{p}\left(\frac{A\alpha\beta}{1+n} + \ell^{-1}k^{1-\alpha}\right) - A^2\alpha\beta(1-\alpha)k^{\alpha}$$

$$\tilde{G}_6 = G_6(A\alpha(1+\beta\ell)) = A\alpha(1+\beta\ell)(1+n)k + \frac{A\alpha\beta\ell^2\bar{a}(1-D\gamma)}{1+n} \tag{24}$$

$$- A^2\alpha\beta\ell(1-\alpha)k^{\alpha} - \frac{\gamma\ell D^2k^{1-\alpha}}{2} + \ell\bar{a}(1-D\gamma)k^{1-\alpha} + A\gamma\ell Dk(1+\alpha\beta\ell)$$

$$\text{where } D = (A(1-\alpha)k^{\alpha} - \bar{a}) \tag{25}$$